Conditional McKean-Vlasov SDEs with Jumps and Regime Switching and Associated Systems of Nonlocal Partial Integro-Differential Equations

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Abstract

We study a class of conditional McKean-Vlasov stochastic differential equations (SDEs) that incorporate both jumps and regime-switching dynamics, aiming to model complex systems subject to sudden shocks and changes in their environment. This class of models extends classical mean-field models to capture richer, real-world phenomena such as discontinuous shocks and environment-driven randomness. The dynamics describe systems in which the evolution of each particle depends not only on its individual state and various types of noise, but also on the conditional law of the entire system given a filtration − a natural framework for modeling partial information, systemic risk, and interacting agents under uncertainty.

We provide a detailed analysis of the well-posedness, stability, and regularity properties of these systems, highlighting the intricate interplay between Poisson jumps, Markovian regime switching, and the conditional distribution. In particular, we examine the first- and second-order derivatives of the solution with respect to the probability law, employing the framework of Lions' derivatives on the Wasserstein space.

Moreover, we show that these stochastic systems are naturally associated to a novel class of nonlinear, nonlocal systems of partial integro-differential equations (PIDEs) with terminal conditions. Under suitable assumptions, we establish a probabilistic representation of their unique classical solutions via an extended Feynman-Kac-type formula.

The results establish a rigorous bridge between stochastic particle systems under symmetric information and their analytical counterparts, offering new insights for applications in finance, neuroscience, and controlled network systems.