Khovanov Homology of the $\mathbf{P(\pm l, m, n)}$ pretzel links

In [Kho] Mikhail Khovanov defined, for a given diagram of an oriented link $\vec{L}$, a collection of groups $\mathcal{H}^{i,j}(\vec{L})$ numerated by pairs of integers. These Khovanov groups were constructed as cohomology groups of certain chain complexes, where it turned out that the Euler characteristic of these complexes gave a version of the Jones polynomial of the link. Subsequently, Oleg Viro in [Vir1, Vir2], reformulated, for non-oriented framed links, Khovanov's definitions and obtained "friendlier" Khovanov homology groups $H_{a,b}(L)$, where L is a non-oriented framed link. The classical Khovanov cohomology $\mathcal{H}^{i,j}(\vec{L})$, and the framed version of Khovanov homology $H_{a,b}(L)$, are related by the following:

$$\mathcal{H}^{i,j}(\vec{L}) \cong H_{w-2i,3w-2j}(L) \cong H_{a,b}(L) \cong \mathcal{H}^{\frac{w-a}{2},\frac{3w-b}{2}}(\vec{L}),$$

where $w(\vec{L}) = w$ is the writhe of the oriented link diagram $\vec{L}$.

We use Viro's approach to compute $H_{a,b}(L)$ for the pretzel links of the type $L = P(l, m, n)$ and $L = P(-l, m, n)$. The computation involves using the long exact sequence of Khovanov homology and the Khovanov homology groups of torus links of type $T(2, n)$ which were computed in [MV2]. This is part of a joint work done with Gabriel Montoya-Vega.

Keywords: Jones polynomial, Khovanov homology, framed links

References

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