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Crossing numbers of complete tripartite and balanced complete multipartite graphs
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Abstract

The *crossing number* $\mathrm{cr}(G)$ of a graph G is the minimum number of crossings in a nondegenerate planar drawing of G. A nondegenerate planar drawing of G is *rectilinear* if every edge is drawn as a straight line segment, and the *rectilinear crossing number* $\overline{\mathrm{cr}}(G)$ of G is the minimum number of crossings in a rectilinear drawing of G; clearly the rectilinear crossing number of G is an upper bound for the crossing number of G. Zarankiewicz defined a bound $Z(n_1, n_2)$, proved that $\overline{\mathrm{cr}}(K_{n_1,n_2}) \leq Z(n_1, n_2)$ and attempted to prove $\overline{\mathrm{cr}}(K_{n_1,n_2}) = Z(n_1, n_2)$; the latter equality has become known as Zarankiewicz's Conjecture. Artist Anthony Hill defined $H(n)$ and produced drawings of K_n that realize $H(n)$ crossings; it is has been shown that $\mathrm{cr}(K_n) \leq H(n)$ and conjectured that $\mathrm{cr}(K_n) = H(n)$, but it is known that $\overline{\mathrm{cr}}(K_n) < \mathrm{cr}(K_n)$. There has been extensive study of crossing numbers and rectilinear crossing numbers of complete bipartite graphs and complete graphs. A brief history of the crossing numbers of these graphs will be presented.

Much less is known for complete tripartite graphs, and more generally for complete multipartite graphs. An analogous bound $A(n_1, n_2, n_3)$ is defined for the complete tripartite graph and the following results are presented: $\overline{\mathrm{cr}}(K_{n_1,n_2,n_3}) \leq A(n_1, n_2, n_3)$, and for n sufficiently large, $0.973A(n, n, n) \leq \overline{\mathrm{cr}}(K_{n,n,n})$ and $0.666A(n, n, n) \leq \mathrm{cr}(K_{n,n,n})$.

A complete multipartite graph is balanced if the partite sets all have the same cardinality. Asymptotic behavior of the crossing number of the balanced complete r-partite graph is discussed. Richter and Thomassen proved that the limit as $n \to \infty$ of $\mathrm{cr}(K_{n,n})$ over the maximum number of crossings in a drawing of $K_{n,n}$ exists and is at most $\frac{1}{4}$. For a fixed r and the balanced complete r-partite graph, $\zeta(r) := \frac{3(r^2-r)}{8(r^2+r-3)}$ is an upper bound to the limit superior of the crossing number divided by the maximum number of crossings in a drawing.

Wednesday, February 25, 2015
10:00 am
C-202