Abstract

We construct Cayley graphs $\Gamma(G, S)$ of the projective special linear group $G = \mathrm{PSL}_2(\mathbb{F}_q)$, for prime q and $2 \leq q \leq 31$, and some of their symmetric subsets $S \subset G$. Certain properties of these graphs, such as girth, diameter, bipartite-ness, connectedness, and Ramanujancy, are determined. We classify the number of these graphs up to isomorphism, given a pair of girth and diameter. The isomorphism is carried out via implementations and existing algorithms in the software SAGE.

Our research contribution of the thesis is the discovery of some graphs that have very high girth (some matching the best known cubic Cayley graphs and some of degree 4 and 5). We also discover a large number of Ramanujan graphs in this process. These properties make these graphs quite suitable for applications. For concrete applications, these graphs are described and are used to construct edge-vertex incidence graphs Γ_B. We use these graphs Γ_B to construct low-density parity-check codes (LDPC). Then we carry out simulations for their error-correcting capability.

The bipartite graphs Γ_B are used to construct the parity-check matrix $H_{\mathscr{C}_{q,S}}$ of some linear codes $\mathscr{C}_{q,S}$. Certain properties of these codes, such as length, dimension, rate, and minimum distance, are studied. We compare the performance of some of these codes through the binary symmetric channel (BSC) considering different values of the crossover probability p.

We provide examples of graphs and codes.