

An extension of the p -adic integers

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Abstract

For every element in the Kleene set A^* of finite words generated from the alphabet $A = \{0, 1, \dots, p-1\}$, we can do a one-to-one association with the nodes in a complete p -ary tree and identify these nodes by the corresponding word. In this work we associate every node n (or equivalently, word in A^*) in the complete p -ary tree with an element x in the p -adic integers $\mathbb{Z}_p$, and say that the pair (n, x) , is a p -adic node. Any finite multiset of p -adic integer nodes is defined as an extended p -adic integer and the set of these multisets is denoted by $\mathbb{P}_p$. Various equivalencies are defined on $\mathbb{P}_p$, as well addition and multiplication in such way that $\mathbb{P}_p$ is a $\mathbb{Z}_p$ -module and there are monomorphisms on these operations between the p -adic integers $\mathbb{Z}_p$ and $\mathbb{P}_p$. Complete p -ary trees are used to illustrate the extended p -adic integers and these operations. An application we have found for the extended p -adic integer is the solution of polynomial p -adic equations with a method that use extended p -adic integer trees (EpAITM). This method could be a good alternative to Hensel's Lemma when it does not apply.