



UNIVERSITY OF PUERTO RICO AT RIO PIEDRAS

---

**NSF-CBMS CONFERENCE:**

**K-THEORY OF OPERATOR  
ALGEBRAS AND ITS  
APPLICATIONS TO GEOMETRY  
AND TOPOLOGY**

---

AUGUST 8-12 2022

Principal Lecturer: Guoliang Yu

Invited speakers:

Simone Cecchini, Robin Deeley, Carla Farsi, Zhuang Niu,  
Yanli Song, Xiang Tang, Hang Wang, Jinmin Wang, Kun  
Wang, Rufus Willett, Jianchao Wu, Zhizhang Xie

Sponsored by



Organizing Committee:

Iván Cardona (University of Puerto Rico, Río Piedras),  
Guihua Gong (University of Puerto Rico, Río Piedras),  
Liangqing Li (University of Puerto Rico, Río Piedras),  
Lin Shan (University of Puerto Rico, Río Piedras),  
Kun Wang (Texas A&M University)



# K-theory of operator algebras and its applications to geometry and topology

---

## Agenda

### Day 1, August 8, 2022

08:15 – 08:30 Welcome (Chancellor/Dean/Chair)

---

08:30 – 09:20 Guoliang Yu

---

09:40 – 10:30 Guoliang Yu

---

10:30 – 11:00 Tea break

---

11:00 – 11:50 Xiang Tang

---

12:00 – 14:00 Lunch break

---

14:00 – 14:50 Rufus Willett

---

15:10 – 16:00 Robin Deeley

---

16:00 – 16:30 Tea break

---

16:30 – 17:20 Simone Cecchini

---

## **Day 2, August 9, 2022**

**08:30 – 09:20 Guoliang Yu**

---

**09:40 – 10:30 Guoliang Yu**

---

**10:30 – 11:00 Tea break**

---

**11:00 – 11:50 Kun Wang**

---

**12:00 – 14:00 Lunch break**

---

**14:00 – 14:50 Carla Farsi**

---

**15:10 – 16:00 Yanli Song**

---

**16:00 – 16:30 Tea break**

---

**16:30 – 16:50 Angel Roman**

---

**17:00 – 17:20 Contributed Talk**

---

## **Day 3, August 10, 2022**

**08:30 – 09:20 Guoliang Yu**

---

**09:40 – 10:30 Guoliang Yu**

---

**10:30 – 11:00 Tea break**

---

**11:00 – 11:50 Hang Wang**

---

**12:00 – 14:00 Lunch break**

---

## **Day 4, August 11, 2022**

**08:30 – 09:20   Guoliang Yu**

---

**09:40 – 10:30   Guoliang Yu**

---

**10:30 – 11:00   Tea break**

---

**11:00 – 11:50   Zhuang Niu**

---

**12:00 – 14:00   Lunch break**

---

**14:00 – 14:50   Zhizhang Xie**

---

**15:10 – 16:00   Jinmin Wang**

---

**16:00 – 16:30   Tea break**

---

**16:30 – 16:50   Contributed Talk**

---

**17:00 – 17:20   Contributed Talk**

---

## **Day 5, August 12, 2022**

**08:30 – 09:20 Guoliang Yu**

---

**09:40 – 10:30 Guoliang Yu**

---

**10:30 – 11:00 Tea break**

---

**11:00 – 11:50 Jianchao Wu**

---

**12:00 – 14:00 Lunch break**

---

**14:00 – 14:20 Contributed Talk**

---

**14:30 – 14:50 Contributed Talk**

---

**15:10 – 15:30 Contributed Talk**

---

**15:40 – 16:00 Contributed Talk**

---

**16:00 – 16:30 Tea break**

---

**16:30 – 16:50 Contributed Talk**

---

**17:00 – 17:20 Contributed Talk**

---

# Contents

<b>Introduction</b>	<b>1</b>
<b>Monday, August 8, 2022</b>	<b>5</b>
Lecture 1: K-theory of operator algebras ( <i>Guoliang Yu</i> ) . . . . .	6
Lecture 2: Operator algebras from geometry and topology ( <i>Guoliang Yu</i> ) . . . . .	7
Helton-Howe Trace, Connes-Chern Character, and Quantization ( <i>Xiang Tang</i> ) . . . . .	8
Controlled KK-theory, decompositions, and the Universal Coefficient Theorem ( <i>Rufus Willett</i> ) . . . . .	9
Smale spaces and dynamic asymptotic dimension ( <i>Robin Deeley</i> ) . . . . .	10
A long neck principle for Riemannian spin manifolds with positive scalar curvature ( <i>Simone Cecchini</i> ) . . . . .	11
<b>Tuesday, August 9, 2022</b>	<b>13</b>
Lecture 3: Localization algebras and K-homology ( <i>Guoliang Yu</i> ) . . . . .	14
Lecture 4: Evaluations and the Baum-Connes assembly map ( <i>Guoliang Yu</i> ) . . . . .	15
About Cuntz semigroup and Cuntz comparison for C*-algebras ( <i>Kun Wang</i> ) . . . . .	16
Isometries of Kellendonk-Savinien Spectral Triples ( <i>Carla Farsi</i> ) . . . . .	17
Higher APS index theorem for proper Lie group action ( <i>Yanli Song</i> ) . . . . .	18
The Mackey bijection in C*-algebras ( <i>Angel Roman</i> ) . . . . .	19
Title ( <i>Contributed Talk</i> ) . . . . .	20
<b>Wednesday, August 10, 2022</b>	<b>21</b>
Lecture 5: Elliptic operator and their primary invariants ( <i>Guoliang Yu</i> ) . . . . .	22
Lecture 6: Computations and applications of primary invariants ( <i>Guoliang Yu</i> ) . . . . .	23
Topological K-theory for discrete groups and index formula ( <i>Hang Wang</i> ) . . . . .	24
<b>Thursday, August 11, 2022</b>	<b>25</b>
Lecture 7: Secondary invariants of elliptic operators ( <i>Guoliang Yu</i> ) . . . . .	26
Lecture 8: Additivity of higher rho invariants ( <i>Guoliang Yu</i> ) . . . . .	27
Structure of crossed product C*-algebras ( <i>Zhuang Niu</i> ) . . . . .	28
On Gromov's dihedral extremality and rigidity conjectures ( <i>Zhizhang Xie</i> ) . . . . .	29
Dihedral Rigidity Conjecture and Stoker's Problem ( <i>Jinmin Wang</i> ) . . . . .	30
Title ( <i>Contributed Talk</i> ) . . . . .	31
Title ( <i>Contributed Talk</i> ) . . . . .	32
<b>Friday, August 12, 2022</b>	<b>33</b>
Lecture 9: Applications of secondary invariants to geometry and topology ( <i>Guoliang Yu</i> ) . . . . .	34
Lecture 10: Open questions and future development ( <i>Guoliang Yu</i> ) . . . . .	35
The Novikov conjecture and groups of diffeomorphisms ( <i>Jianchao Wu</i> ) . . . . .	36

Title ( <i>Contributed Talk</i> ) . . . . .	37
Title ( <i>Contributed Talk</i> ) . . . . .	38
Title ( <i>Contributed Talk</i> ) . . . . .	39
Title ( <i>Contributed Talk</i> ) . . . . .	40
Title ( <i>Contributed Talk</i> ) . . . . .	41
Title ( <i>Contributed Talk</i> ) . . . . .	42
<b>Author Index</b>	<b>43</b>



# Introduction

## 1. Basic Information

**Brief Program Description:** K-theory is a unifying theme in several important areas of mathematics including operator algebras, geometry, topology, and number theory. K-groups are receptacles for both primary invariants and secondary invariants of elliptic operators. Primary invariants have powerful applications such as rigidity of manifolds while secondary invariants can be used to detect more subtle mathematical phenomena such as non-rigidity of manifolds. Guoliang Yu and his coauthors made several significant contributions on introducing and studying these invariants and obtain important application on rigidity of manifolds such as Borel Conjecture, and non-rigidity of manifolds such as the calculation of the structure group and classification of the positive scalar curvature metrices of a manifold.

The principal lecturer, Professor Guoliang Yu, is one of the main figures in the area of K-theory of operator algebras and applications to geometry and topology. Together with his collaborators, Yu obtained some of the best results on the Novikov conjecture and the stable Borel conjecture. In the process, he introduced important concepts such as Yu's localization algebra, Yu's property A, and geometric complexity. Yu developed quantitative (controlled) K-theory for geometric  $C^*$ -algebras. Quantitative K-theory provides a powerful tool in computing K-theory of operator algebras. Yu was an invited speaker at the International Congress of Mathematicians in 2006. He was in the inaugural class of AMS Fellows and is a Simons Fellow in Mathematics. Yu is among the best expositors in this area of mathematics. He has given numerous invited lecture series including the Oberwolfach Seminar in 2014 and Young Mathematicians in  $C^*$ -algebras at Copenhagen in 2017.

The purpose of this conference is to highlight the recent exciting progress in K-theory of operator algebras and their applications to geometry and topology, and to help graduate students and postdocs learn the fundamentals of this broad and technically difficult subject and navigate to the research frontiers.

## 2. Mathematical Themes

Elliptic differential operators on a compact manifold are Fredholm. Atiyah-Singer's celebrated index theorem computes the Fredholm index of an elliptic differential operator in terms of its symbol. The higher index theory of elliptic differential operators is a powerful generalization of the Fredholm index theory and encodes additional information about certain symmetries of the manifold, i.e., its fundamental group. The higher index of an elliptic operator on a compact manifold lives in the K-theory of the  $C^*$ -algebra associated to its fundamental group. The computation of the higher indices would lead to solutions to important problems in topology and geometry. The Baum-Connes conjecture

is a program of computing the K-theory of (reduced) group  $C^*$ -algebras and higher indices of elliptic operators. The strong Novikov conjecture provides an algorithm for determining when the higher index of an elliptic operator is non-zero using information of its symbol. The strong Novikov conjecture implies the Novikov conjecture on homotopy invariance of higher signatures. The Novikov conjecture is a central problem in differential topology since the classification problem for higher dimensional manifolds can be essentially reduced to the Novikov conjecture by the surgery theory.

Higher indices of elliptic operators are obstructions to invertibility. Higher indices are considered to be the primary invariants since they are invariant under homotopy equivalence. When elliptic operators are invertible, the primary invariants (higher indices) vanish and new secondary invariants can be constructed. These secondary invariants are obstructions to locality of the inverse of the elliptic operators. The secondary invariants serve as powerful tools in measuring the size of moduli spaces of positive scalar curvature metrics and the degree of non-rigidity for manifolds. In the last decade or so, Yu made significant contribution to compute the higher indices and the secondary invariants of elliptic operators. In the process of these work, Yu and his coauthors discovered new applications to rigidity and non-rigidity of manifolds and fascinating new connections to other areas of mathematics such as geometry and topology of manifolds, geometric group theory, expander theory, and non-linear geometry of Banach spaces.

Guoliang Yu and his coauthors introduced a theory of geometric complexity to study K-theory. Roughly speaking, a space (or a group) is said to have finite geometric complexity if there is an algorithm to decompose it into simpler, more manageable pieces. The class of groups with finite geometric complexity is very large and includes all countable subgroups of  $GL(n; K)$  (for any field  $K$ ) and all countable subgroups of almost connected Lie groups. They compute K-theory for spaces (or groups) with finite geometric complexity. As an application, they proved the stable Borel conjecture for closed aspherical manifolds whose fundamental groups have finite decomposition complexity. Recall that the stable Borel conjecture states that, a closed aspherical manifold  $M$  is stably rigid in the sense that if another closed manifold  $N$  is homotopy equivalent to  $M$ , then  $N \times \mathbb{R}^n$  is homeomorphic to  $M \times \mathbb{R}^n$  for some  $n$ . Guentner, Tessera and Yu introduced a theory of dynamic asymptotic dimension to compute K-theory of the crossed product  $C^*$ -algebras. A key computational tool is the quantitative/controlled K-theory.

Higher indices of elliptic operators are examples of primary invariants. The (coarse) Baum-Connes conjecture is an algorithm of computing higher indices. The coarse Baum-Connes conjecture holds for spaces with finite geometric complexity. Computer scientists discovered expanders in their search for networks with high connectivity. Expanders have found surprising applications to group theory, number theory, and ergodic theory. The difficulty with expanders is that they are no longer decomposable in any reasonable sense (e.g. they don't have finite geometric complexity). Guoliang and his coauthors computed the K-theory of the maximal Roe algebra for certain expanders. The result of the computation verifies the maximal coarse Baum-Connes conjecture for these expanders. We use this connection to show that the maximal Baum-Connes conjecture with certain coefficients holds for the Gromov monster groups despite the fact that the Baum-Connes assembly map with the same coefficients is injective but not surjective for these groups. When the higher index of an elliptic operator vanishes, one can define a secondary invariant called the higher rho invariant (discovered by Higson and Roe). This subtle secondary invariant can be used to detect finer geometric and topological structures. Guoliang and coauthors apply the higher rho invariant to measure the size of the moduli space of all positive scalar curvature metrics and the degree of non-rigidity of topological manifolds.

The application to non-rigidity requires the following two important ingredients. First, Yu, jointly with Weinberger and Xie, solves a long standing open question stating that the higher rho invariant is a homomorphism from the structure group of a topological manifold to the K-group of a certain  $C^*$ -algebras. Secondary, Dr. Yu and coauthors develop a notion of (strongly) finitely embeddable groups to obtain a lower bound for K-theory of the maximal group  $C^*$ -algebra. The strong Novikov conjecture provides an algorithm to determine when the higher index of an elliptic operator is nonzero in K-theory of group  $C^*$ -algebras. In a paper of Kasparov and Yu, they proved the strong Novikov conjecture for groups coarsely embeddable into Banach spaces satisfying a geometric condition called Property (H). More recently, joint with Sherry Gong and Jianchao Wu, Yu proved the Novikov conjecture for groups acting properly and isometrically on infinite dimensional non-positively curved spaces. In particular, The Novikov conjecture is true for any discrete subgroup of the volume preserving diffeomorphisms of a compact smooth manifold. Yu proved the algebraic Novikov conjecture for group algebras over the ring of the Schatten class operators for all groups.

There is a well established Puerto Rican mathematical community of mathematicians and PhD students working on K-theory of  $C^*$ -algebra and their classifications. It is our hope that this conference will create not only synergy between different areas of mathematics but also an impact on the local mathematical community.

### 3. Principal Lecturer

The principal lecturer, Professor Guoliang Yu is a university distinguished professor and Powell chair in mathematics at Texas A & M University. Yu is one of the main figures in the area of noncommutative geometry and operator algebras. Together with his collaborators, Yu obtained some of the best results on the Novikov conjecture and the stable Borel conjecture. In the process, he introduced important concepts such as Yu's localization algebra, Yu's property A, and geometric complexity. Yu developed quantitative (controlled) K-theory for geometric  $C^*$ -algebras. Quantitative K-theory provides a powerful tool in computing K-theory of operator algebras. Yu was an invited speaker at the International Congress of Mathematicians in 2006. He was in the inaugural class of AMS Fellows.

Yu is an outstanding speaker. Even though the nature of most of his work is very difficult both conceptually and technically, the talks he has given at conferences are very concise and accessible. Yu is among the best expositors in this area of mathematics. The EMS text book of Nowak and Yu is now considered by many, one of the best books in K-theory with applications to Geometry and Topology. Willett and Yu have recently written a book on higher index theory. He has given numerous invited lecture series including the Oberwolfach Seminar in 2014 and Young Mathematicians in  $C^*$ -algebras at Copenhagen in 2017.



**Monday, August 8, 2022**

---

## Lecture 1: K-theory of operator algebras

Guoliang Yu  
Texas A&M University  
guoliangyu@tamu.edu

July 10, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

### Abstract

This lecture will be devoted to an overview of K-theory for operator algebras. Particular emphasis will be given to the index map in the six term exact sequence of K-theory for operator algebras. No prior knowledge of operator K-theory is assumed.

Monday  
August 8  
8:30am

---

## Lecture 2: Operator algebras from geometry and topology

Guoliang Yu  
Texas A&M University  
guoliangyu@tamu.edu

July 10, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

### Abstract

In this lecture, the speaker will introduce  $C^*$ -algebras which naturally arise from geometry and topology. Examples of such  $C^*$ -algebras include group  $C^*$ -algebras and Roe algebras associated to metric spaces.

Monday  
August 8  
9:40am

---

# Helton-Howe Trace, Connes-Chern Character, and Quantization

Xiang Tang  
Washington University in St. Louis  
xtang@wustl.edu

July 9, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

## Abstract

Monday  
August 8  
11:00am

In the early 70s, Helton and Howe proved a beautiful formula for the trace of commutators of Toeplitz operators. In the 80s, Connes greatly generalized the Helton-Howe trace formula using cyclic cohomology. The Connes-Chern character contains the Helton-Howe trace as the top degree component. In this talk, we will study the Connes-Chern character for the Toeplitz extension from the viewpoint of quantization. As an outcome, we will establish the Helton-Howe trace formula for Toeplitz operators with  $C^2$ -symbols for all weighted Bergman spaces. This talk is based on joint work with Yi Wang and Dechao Zheng.

*Keywords:* Connes-Chern character, Helton-Howe trace, quantization, Toeplitz operator, weighted Bergman space

## References

- [1] Alain Connes. Noncommutative differential geometry. *Inst. Hautes Études Sci. Publ. Math.*, (62):257–360, 1985.
- [2] Alain Connes. *Noncommutative geometry*. Academic Press, Inc., San Diego, CA, 1994.
- [3] J. William Helton and Roger E. Howe. Integral operators: commutators, traces, index and homology. In *Proceedings of a Conference Operator Theory (Dalhousie Univ., Halifax, N.S., 1973)*, pages 141–209. Lecture Notes in Math., Vol. 345, 1973.
- [4] J. William Helton and Roger E. Howe. Traces of commutators of integral operators. *Acta Math.*, 135(3-4):271–305, 1975.
- [5] X. Tang, Y. Wang, and D. Zheng, Helton-Howe Trace, Connes-Chern Character and Quantization, *arXiv:2204.04337*.



---

# Controlled KK-theory, decompositions, and the Universal Coefficient Theorem

Rufus Willett  
University of Hawaii  
rufus@math.hawaii.edu

July 11, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

## Abstract

I will discuss ‘controlled KK-theory’, which gives a new way of making sense of KK-classes ‘locally’ with respect to subsets of a noncommutative  $C^*$ -algebra. I will then reformulate the Universal Coefficient Theorem (UCT) in this context, and give new sufficient conditions for the UCT to hold.

This is based mainly on joint work with Guoliang Yu, and also on some joint work with Arturo Jaime.

Monday  
August 8  
2:00pm

---

## Smale spaces and dynamic asymptotic dimension

Robin Deeley  
University of Colorado Boulder  
robin.deeley@colorado.edu

July 10, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

### Abstract

I will discuss joint work with Karen Strung. We show that the stable, unstable, and homoclinic  $C^*$ -algebras associated to a Smale spaces have finite nuclear dimension. Our proof of finite nuclear dimension relies on Guentner, Willett, and Yu's notion of dynamic asymptotic dimension. No knowledge of Smale spaces is required for this talk.

Monday  
August 8  
3:10pm

---

# A long neck principle for Riemannian spin manifolds with positive scalar curvature

Simone Cecchini  
Texas A&M  
cecchinisimone1@gmail.com

July 11, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

## Abstract

I will present results in index theory on compact Riemannian spin manifolds with boundary in the case when the topological information is encoded by bundles which are supported away from the boundary. As a first application, we establish a “long neck principle” for a compact  $n$ -dimensional Riemannian spin manifold with boundary  $X$ , stating that if  $\text{scal}(X) \geq n(n-1)$  and there is a non-zero degree map  $f$  into the  $n$ -sphere which is area decreasing, then the distance between the support of the differential of  $f$  and the boundary of  $X$  is at most  $\pi/n$ . This answers, in the spin setting, a question asked by Gromov. As a second application, we consider a manifold  $X$  obtained by removing  $k$  pairwise disjoint embedded  $n$ -balls from a closed  $n$ -dimensional spin manifold  $Y$ . We show that if  $\text{scal}(X) > \sigma > 0$  and  $Y$  satisfies a certain  $K$ -theoretic condition, then the width of a geodesic collar neighborhood of  $\partial X$  is at most  $\pi\sqrt{(n-1)/(n\sigma)}$ . This applies, for example, when  $Y$  is the  $n$ -torus. I will also discuss, in the same setting, refined metric inequalities in the case when the mean curvature of the boundary is taken into account. The results involving mean curvature are joint work with Rudolf Zeidler.

Monday  
August 8  
4:30pm

*Keywords:* Scalar curvature, spin geometry, metric inequalities.

## References

- [1] S. Cecchini, A long neck principle for Riemannian spin manifolds with positive scalar curvature, *Geometric and Functional Analysis* 30, No.5 (2020) 1183–1223.
- [2] S. Cecchini, R. Zeidler, Scalar and mean curvature comparison via the Dirac operator, *Geometry & Topology (to appear)*. <https://msp.org/soon/coming.php?jpath=gt>  
*arXiv preprint*. <https://arxiv.org/pdf/2103.06833.pdf>



**Tuesday, August 9, 2022**

---

## Lecture 3: Localization algebras and K-homology

Guoliang Yu  
Texas A&M University  
guoliangyu@tamu.edu

July 10, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

### Abstract

In this lecture, the speaker will introduce the concept of localization algebra and use its K-theory to present an elementary description of K-homology theory.

Tuesday  
August 9  
8:30am

---

## Lecture 4: Evaluations and the Baum-Connes assembly map

Guoliang Yu  
Texas A&M University  
guoliangyu@tamu.edu

July 10, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

### Abstract

In this lecture, the speaker will use the evaluation map from the localization algebra to construct the Baum-Connes map. The advantage of this approach is that this construction uses only elementary operator K-theory and accessible to beginners.

Tuesday  
August 9  
9:40am

---

## About Cuntz semigroup and Cuntz comparison for $C^*$ -algebras

Kun Wang  
Texas A&M University  
kunwangmath@gmail.com

July 10, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

### Abstract

Tuesday  
August 9  
11:00am

Examples due to Villadsen, Rørdam, and Toms have shown that Elliott invariant is insufficient for the classification of all simple, separable, and nuclear  $C^*$ -algebras. There are simple, separable, and nuclear  $C^*$ -algebras that can be distinguished by their Cuntz semigroups but not by their Elliott invariant.

In this talk, I will introduce the definition and some properties of the Cuntz semigroup. In [1], Brown, Perera, and Toms recovered the Cuntz semigroup for a well behaved class of simple  $C^*$ -algebras by using the ingredients of the Elliott invariant—the Murray-von Neumann semigroup and the cone of lower semicontinuous traces. (See also in [3], in which the non-unital case was considered.) In this talk, we give a characterization of the Cuntz comparison for a class of  $C^*$ -algebras with one non-trivial closed two-sided ideal, by using the Murray-von Neumann semigroup and the cone of lower semicontinuous traces.

*Keywords:* Classification,  $C^*$ -algebra, Cuntz semigroup, K-theory, trace

### References

- [1] N. P. Brown, F. Perera, A. S. Toms, The Cuntz semigroup, the Elliott conjecture, and dimension functions on  $C^*$ -algebras, *J. Reine Angew. Math.* 621 (2008), 191-211.
- [2] K. Wang, Equivalence of two Invariants of  $C^*$ -algebras with the ideal property, *J. Non-commut. Geom.* 12 (2018), 1-27.
- [3] G. A. Elliott, L. Robert and L. Santiago, *The cone of lower semicontinuous traces on a  $C^*$ -algebra*, *Amer. J. Math.* 133 (4) (2011), 969-1005.



---

# Isometries of Kellendonk-Savinien Spectral Triples

Carla Farsi  
University of Colorado  
carla.farsi@colorado.edu

July 9, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

## Abstract

In this talk, I will first review the definitions and properties of the large and small isometry groups associated to a spectral triple. These generalize classical isometry groups. For example, for the standard spectral triple associated to manifold, both of these isometry groups are equal to the isometry group of the manifold.

However much interest has been recently centered on noncommutative cases.

In the particular case of the spectral triples of Kellendonk-Savinien [2] on ultrametric Cantor sets, Conti and I proved in [1] that the small and large isometry groups coincide. In the special case of binary trees, these isometry groups are equal to  $\mathbb{Z}$ , thus generalizing the Cantor set isometry group situation.

*Keywords:* spectral triples, isometries

Tuesday  
August 9  
2:00pm

## References

- [1] R. Conti and C. Farsi, Isometries of Kellendonk-Savinien Spectral Triples and Connes metrics, submitted.
- [2] J. Kellendonk and J. Savinien, Spectral triples and characterization of aperiodic order, Proc. Lond. Math. Soc. (3) 104 (2012), 123–157.

---

## Higher APS index theorem for proper Lie group action

Yanli Song  
Washington University in St. Louis  
yanlisong@wustl.edu

July 10, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

### Abstract

Tuesday  
August 9  
3:10pm

In this talk, we will present an index theorem for proper cocompact Lie group actions on manifolds with boundary, which generalizes the Atiyah- Patodi-Singer index theorem for compact manifolds and the Atiyah-Bott fixed point index theorem for compact Lie group actions on closed manifolds. In addition, We give sufficient conditions ensuring the well-definedness of the delocalized eta invariant associated to a Dirac operator on a cocompact manifold without boundary. The talk is based on the joint work with Paolo Piazza, Hessel Posthuma and Xiang Tang.

---

## The Mackey bijection in $C^*$ -algebras

Angel Roman  
Washington University in St. Louis  
romanmartinez.angel@gmail.com

July 25, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

### Abstract

Abstract: The Mackey bijection is a one-to-one correspondence between tempered irreducible unitary representations of reductive groups and the irreducible unitary representations of their associated Cartan motion groups. In this talk, I will present an embedding of the reduced group  $C^*$ -algebra of the motion group into the reduced group  $C^*$ -algebra of the semisimple group. I will present an overview of the construction of the embedding using continuous fields of  $C^*$ -algebras.

Friday  
August 9  
4:30pm

---

Your title here

Author Name Here  
University Here  
emailhere@wustl.edu

July 9, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

## Abstract

TBA

*Keywords:* Here

Tuesday  
August 9  
5:00pm

**Wednesday, August 10, 2022**

---

## Lecture 5: Elliptic operator and their primary invariants

Guoliang Yu  
Texas A&M University  
guoliangyu@tamu.edu

July 10, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

### Abstract

In this lecture, the speaker will introduce the concepts of elliptic operators and their primary invariants: higher indices. Emphasis will be given to geometric elliptic operators such as the Dirac operators and signature operators.

Wednesday  
August 10  
8:30am

---

## Lecture 6: Computations and applications of primary invariants

Guoliang Yu  
Texas A&M University  
guoliangyu@tamu.edu

July 10, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

### Abstract

In this lecture, the speaker will compute the primary invariants of geometric elliptic operators in interesting cases and give the applications of these higher invariants to geometry and topology such as the Novikov conjecture and the Gromov-Lawson-Rosenberg conjecture.

Wednesday  
August 10  
9:40am

---

# Topological K-theory for discrete groups and index formula

Hang Wang  
East China Normal University  
wanghang@math.ecnu.edu.cn

July 10, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

## Abstract

Wednesday  
August 10  
11:00am

Topological K-theory of a discrete group was introduced in the original formulation of the Baum-Connes conjecture and was later identified as the equivalent K-homology of the universal space of proper actions by the group. As the left hand side of the Baum-Connes conjecture, it displays local feature of an associated elliptic operator, while the right hand side of Baum-Connes accommodates higher indices and displays global feature of the operator. In this lecture we will review various notions of K-homology and introduce a pairing of the topological K-theory with cyclic cocycles coming from the group algebra and the local index formula computing the pairing. This is joint work with Paulo Carrillo-Rouse and Bai-Ling Wang.



**Thursday, August 11, 2022**

---

## Lecture 7: Secondary invariants of elliptic operators

Guoliang Yu  
Texas A&M University  
guoliangyu@tamu.edu

July 10, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

### Abstract

In this lecture, the speaker will introduce the concept of the secondary invariants discovered by Higson-Roe when the primary invariant of an elliptic operator vanishes.

Thursday  
August 11  
8:30am

---

## Lecture 8: Additivity of higher rho invariants

Guoliang Yu  
Texas A&M University  
guoliangyu@tamu.edu

July 10, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

### Abstract

In this lecture, the speaker will outline a proof for additivity of the secondary invariants for certain relative signature operator and explain the significance of this result to topology.

Thursday  
August 11  
9:40am

---

## Structure of crossed product $C^*$ -algebras

Zhuang Niu  
University of Wyoming  
zniu@uwyo.edu

July 10, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

### Abstract

Consider a dynamical system, and let us study the structure of the corresponding crossed product  $C^*$ -algebra, in particular on the classifiability, comparison, and stable rank. More precisely, let us introduce a uniform Rokhlin property and a relative comparison property (these two properties hold for all free and minimal  $\mathbb{Z}^d$  actions, and it is plausibly that they hold for all free and minimal actions by an arbitrary amenable group). With these two properties, the crossed product  $C^*$ -algebra is shown to always have stable rank one, to satisfy the Toms-Winter conjecture, and that the comparison radius is dominated by half of the mean dimension of the dynamical system. If time permits, let us also discuss  $C^*$ -dynamical systems and groupoids.

Thursday  
August 11  
11:00am

---

## On Gromov's dihedral extremality and rigidity conjectures

Zhizhang Xie  
Texas A&M University  
xie@math.tamu.edu

July 10, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

### Abstract

Gromov's dihedral extremality and rigidity conjectures concern comparisons of scalar curvature, mean curvature and dihedral angle for manifolds with corners (and more generally manifolds with polyhedral boundary). They have very interesting consequences in geometry and mathematical physics. The conjectures themselves can in some sense be viewed as “localizations” of the positive mass theorem. In this talk, I will explain some recent work on positive solutions to these conjectures. The talk is based on my joint papers with Jinmin Wang and Guoliang Yu.

Thursday  
August 11  
2:00pm

---

## Dihedral Rigidity Conjecture and Stoker's Problem

Jinmin Wang  
Texas A& M University  
jinmin@tamu.edu

July 10, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

### Abstract

Thursday  
August 11  
3:10pm

The Stoker Conjecture states that the dihedral angles of a convex Euclidean polyhedron completely determine the angles of each face. In this talk, I will present my recent work joint with Zhizhang Xie and Guoliang Yu that answers positively to the Stoker Conjecture in all dimensions. Our work proves a more general dihedral rigidity theorem, which concerns the comparison of scalar curvature, mean curvature, and dihedral angles for convex polyhedrons, or more general, manifolds with polytope boundary. We use index theory on manifolds with polytope boundary and the Dirac operator methods.

---

Your title here

Author Name Here  
University Here  
emailhere@wustl.edu

July 9, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

**Abstract**

TBA

*Keywords:* Here

Thursday  
August 11  
4:30pm

---

Your title here

Author Name Here  
University Here  
emailhere@wustl.edu

July 9, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

**Abstract**

TBA

*Keywords:* Here

Thursday  
August 11  
5:00pm



**Friday, August 12, 2022**

---

## Lecture 9: Applications of secondary invariants to geometry and topology

Guoliang Yu  
Texas A&M University  
guoliangyu@tamu.edu

July 10, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

### Abstract

Friday  
August 12  
8:30am

In this lecture, the speaker will discuss the applications of the secondary invariants to non-rigidity of topological manifolds and the moduli space of the Riemannian metrics with positive scalar curvatures.

---

## Lecture 10: Open questions and future development

Guoliang Yu  
Texas A&M University  
guoliangyu@tamu.edu

July 10, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

### Abstract

In the last lecture, the speaker plans to discuss a variety of open questions and future directions.

Friday  
August 12  
9:40am

---

# The Novikov conjecture and groups of diffeomorphisms

Jianchao Wu  
Fudan University  
jianchao\_wu@fudan.edu.cn

July 10, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

## Abstract

The Novikov conjecture is a prominent problem in differential topology. The operator  $K$ -theoretic approach provided by noncommutative geometry has yielded some of the best results that verify this conjecture for vast classes of groups. A natural class of groups for which the conjecture remains largely mysterious is that of countable groups of diffeomorphisms on smooth manifolds. In an upcoming joint paper with Sherry Gong, Zhizhang Xie, and Guoliang Yu, we prove that the (rational strong) Novikov conjecture holds for geometrically discrete countable subgroups of the group of diffeomorphisms of any closed smooth manifold. This removes the volume-preserving condition in a previous joint paper with S. Gong and G. Yu. At a more technical level, we verify the conjecture for any countable group admitting an isometric and proper action on what we call an admissible continuous field of Hilbert-Hadamard spaces, which are (possibly infinite-dimensional) nonpositively curved spaces.

Friday  
August 12  
11:00am

---

Your title here

Author Name Here  
University Here  
emailhere@wustl.edu

July 9, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

**Abstract**

TBA

*Keywords:* Here

Friday  
August 12  
2:00pm

---

Your title here

Author Name Here  
University Here  
emailhere@wustl.edu

July 9, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

**Abstract**

TBA

*Keywords:* Here

Friday  
August 12  
2:30pm

---

Your title here

Author Name Here  
University Here  
emailhere@wustl.edu

July 9, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

**Abstract**

TBA

*Keywords:* Here

Friday  
August 12  
3:10pm

---

Your title here

Author Name Here  
University Here  
emailhere@wustl.edu

July 9, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

## Abstract

TBA

*Keywords:* Here

Friday  
August 12  
3:40pm



---

Your title here

Author Name Here  
University Here  
emailhere@wustl.edu

July 9, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

**Abstract**

TBA

*Keywords:* Here

Friday  
August 12  
4:30pm

---

Your title here

Author Name Here  
University Here  
emailhere@wustl.edu

July 9, 2022



K-THEORY OF OPERATOR  
ALGEBRAS AND ITS APPLICATIONS  
TO GEOMETRY AND TOPOLOGY

**Abstract**

TBA

*Keywords:* Here

Friday  
August 12  
5:00pm

# Author Index

Cecchini Simone, 11  
Contributed Talk, 20, 31, 32, 37–42  
  
Deeley Robin, 10  
  
Farsi Carla, 17  
  
Niu Zhuang, 28  
  
Roman Angel, 19  
  
Song Yanli, 18

Tang Xiang, 8  
  
Wang Hang, 24  
Wang Jinmin, 30  
Wang Kun, 16  
Willett Rufus, 9  
Wu Jianchao, 36  
  
Xie Zhizhang, 29  
  
Yu Guoliang, 6, 7, 14, 15, 22, 23, 26, 27,  
34, 35