Mathematical Theory of Evolutions Arising in Flow Structure Interactions.

Irena Lasiecka

University of Memphis

Workshop on Dynamics, Control and Numerics for Fractional PDEs
University of Puerto Rico, December 5-7, 2018

Acknowledgments

THANKS:

- National Science Foundation, NSF-DMS 0606682

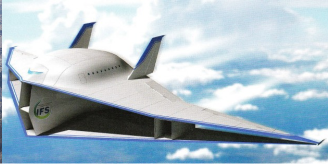
COLLABORATORS:

- Earl Dowell, Duke University.
- Michaela Ignatova, Princeton
- Igor Kukavica, USC, Los Angeles
- Roberto Triggiani, Univ.of Memphis
- Justin Webster, Univ of Maryland
- Igor Chueshov

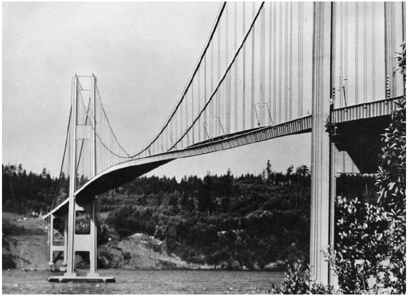
Flow Structure Interactions- Applications

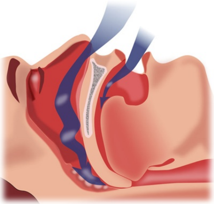
- **Aerodynamics.** Control of **flutter**. Flutter speed.
  - Subsonic, supersonic and transonic regimes;
- **Large space structures.** Large and thin. Highly oscillatory.
- **Medical Sciences**
  - Human respiration- minimize palatal flutter, treatment of apnea
- **Engineering**
  - Oscillating Bridges and Buildings
  - **Harvesting of energy**-Windmills. Post flutter analysis and exchange of energies.

Applications -flow structure supersonic



Applications-flow structure subsonic





Outline

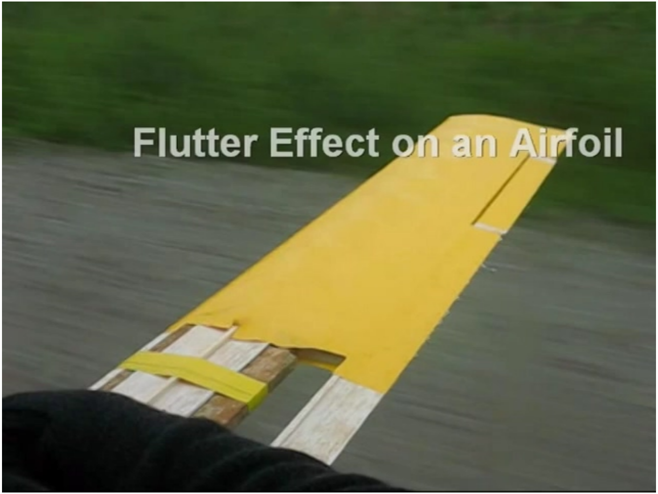
- **Physical goals** -motivation.
- Formulate the problem within **mathematical setting** -identify mathematical problem to study.
- **PDE Models**- Nonlinear Dynamics **Hyperbolic /Hyperbolic-like** with an **Interface.** Euler eq. coupled with plate eq.
- **Role of Modeling** supported by Numerics and Experiment.
- **Main Results**
  - **Representation as a wellposed Dynamical System** (S_t, X).
  - **Stability and long time behavior**
  - **Global attractors.**
  - **Control of the dynamics: stabilization and harvesting**
- Nonlocal Problems **-mixed boundary conditions:Kutta Joukovsky.** Flutter and Finite Hilbert -Riesz Transforms.
- Conclusions and **Open Problems**

Goals and Challenges

- Eliminate the flutter, if possible.
- or control the onset of instability - flutter speed? Where? On the structure.
- post-flutter harvesting. Extract energy from post-onset LCO's

Difficulties: The resulting PDE system has

- No Dissipation
- No Compactness
- Degeneracy of Ellipticity in the Energy Function.[Supersonic]
- Mixed -**nonlocal**- Boundary Conditions.


Flutter Effect on an Airfoil

Question: How to formulate a mathematical question pertinent to these physical problems?

Answer: Given PDE model: study the following:

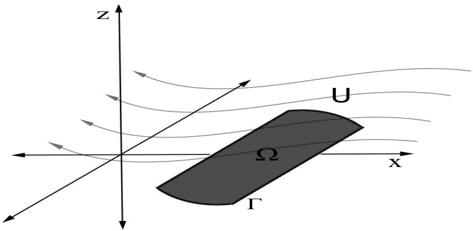
- **Generation** of a nonlinear dynamical system (S_t, X).
- Strong **stability** to equilibria
- Uniform **attraction** of evolution to a **finite dimensional set.**
- **Control** the resulting finite dimensional dynamical system -often **chaotic**.
- **Harvesting energy**: post-flutter analysis - LOC-Limit Oscillating Cycles.

PDE model

- **PDE Model-Flow / Structure Interaction.**
  - E.H. Dowell **Aeorelasticity of plates and shells**, Nordhof 1975, 2004
  - V. Bolotin, *Nonconservative Problems of Elasticity* , Pergamon,1963
  - J.D Cole, L.P. Cook, *Transonic aerodynamics, North Holland, 1986*
  - A.V. Balakrishnan, *Aeroelasticity; Continuum Theory* , Springer 2012.
- **Experimental studies** -wind tunnel:
  NASA Lab at UCLA ,
  AFOSR/NASA Workshop at UCLA 2011,
  Earl H. Dowell and his group, Duke Univ.
- **Numerical studies:**
  - E. Dowell and his group at Duke.
  - J. Howell, Carnegie Mellon.
  - F. Gazzola, Politecnico di Milano

PDE Model.

- Thin, flexible plate, **moving with a velocity** U ($U = 1$ normalised speed of sound).
- Ω is a closed, two-dimensional domain (smooth) in the x-y plane.
- The unperturbed flow is in the negative x-direction.



$\phi(x, y, z; t)$ - **velocity potential at a point,** $u(x, y, t)$ **vertical displacement.**

Structure

Structural equation-$u(t,x,y)$

$$u_{tt} + \Delta^2 u + \mathbf{f}(\mathbf{u}) = p_0 + (\partial_t + U\partial_x)\phi\big|_{\Omega} \ \ in\ \Omega$$
$$Boundary\ \ conditions : BC(u) = Clamped\ \ on\ \partial\Omega$$

Initial conditions: $u(0) = u_0 \in H_0^2(\Omega), u_t(0) = u_1 \in L_2(\Omega)$

$p_0 \in L_2(\Omega)$ is a static aerodynamic pressure on the plate surface.

$f(u)$ is the nonlinearity -internal force $f(u) = -[\mathcal{F}(u), u]$, $\mathcal{F}(u)$ Airy's stress.

aeroelastic potential $= tr_U(\phi) = (\partial_t + U\partial_x)\phi|_{\Omega}$,

Flow

Compressible, Navier Stokes in density ρ,velocity $\mathbf{v}$ and pressure p.
Isentropic, inviscid flow linearize around U. Velocity potential $\nabla\phi = \mathbf{v}$.

Flow equation -$\phi(t,x,y,z)$

$$(\partial_t + U\partial_x)^2\phi = \Delta\phi - \partial_x\phi\partial_x^2\phi \ \ in \ R^3_+$$

$$BC: \ \ on \ \Omega: \ \ \partial_\nu\phi\big|_{z=0} = -(\partial_t + U\partial_x)u(x,y)) \ \ on \ \Omega$$

$$Outside \ \Omega: \partial_\nu\phi\big|_{z=0} = 0 \ \ OR \ (\partial_t + U\partial_x)\phi = 0, \ \ K-J.$$

$$Initial \ \ conditions: \ \ \phi(t=0) = \phi_0, \ \ \phi_t(t=0) = \phi_1$$

- aeroelastic potential $= tr_U(\phi) = (\partial_t + U\partial_x)\phi|_\Omega$,
- downwash $= tr_U(u) = (\partial_t + U\partial_x)u$

PDE -system

A nonlinear plate and perturbed wave, coupled at the interface $\Omega \subset \mathbb{R}^2$:

$$\begin{cases} u_{tt} + \Delta^2 u + \mathbf{f}(\mathbf{u}) = p_0 + tr_U(\phi)|_\Omega \ in\ \Omega \\ BC(u) = Cantilever\ on\ \partial\Omega \\ (\partial_t + U\partial_x)^2\phi = \Delta\phi - \partial_x\phi\partial_x^2\phi\ in\ R^3_+ \\ \partial_\nu\phi\big|_{z=0} = -tr_U(u)(x,y))\ on\ \Omega\ and\ tr_U(\phi) = 0\ outside\ \Omega \\ u(t=0) = u_0,\ u_t(t=0) = u_1;\ \phi(t=0) = \phi_0,\ \phi_t(t=0) = \phi_1 \end{cases}$$

aeroelastic potential, downwash

$f(u) = -[\mathcal{F}(u), u]$ **.**

Two **hyperbolic -like** dynamics coupled on the interface Ω.

Von Karman -Airy Stress function nonlinearity

- $f(u) = -[u, \mathcal{F}(u) - F_0]$. F_0 is in-plane loading .
- $[g, h] = g_{xx}h_{yy} + g_{yy}h_{xx} - 2g_{xy}h_{xy}$ is the von Karman bracket.
- $\mathcal{F}(u)$ is the Airy Stress function, solves

$$\begin{cases} \Delta^2 v = -[u, u] & \text{in } \Omega \\ \mathcal{F}(u) = \frac{\partial \mathcal{F}(u)}{\partial \nu} = 0 & \text{on } \Gamma \end{cases}$$

$f(u)$ cubic, *nonlocal* $f : H^2(\Omega) \to H^{-\epsilon}(\Omega)$ large deflections

Nonlinearity in the model is critical for the analysis, particularly post-flutter analysis.

Goals:

Generation of dynamical system

Stabilization and Control

Expectations: based on experimental studies see - Dowell-McHugh 2016, Kasemi 2018, Tang 2016,Dowell- Saga 2019

- Elimination of the **flutter at the subsonic level** $U < 1$
- Asymptotic **reduction of structural dynamics to a FD attracting sets** (chaotic). **Harvesting** the energy from LCO's.

Mathematical challenges:

- lack of active dissipation on the flow and the structure;
- lack of compactness/regularity;
- potential degeneracy of the energy function.
- Mixed boundary conditions

Work done in the past

- Wellposedness of **finite energy** solutions:
  - **Regularized Models**- containing inertial-rotational forces, $\gamma \Delta u_{tt}$ added. Implies $u_t \in C(H^1(\Omega))$
  - **Strongly damped** plate equation: **regularizing effect is parabolic** : Δu_t added to the model. Implies $u_t \in L_2(H^1(\Omega))$

In all these cases $u_t \in H^1(\Omega)$ simplifies the analysis. However **the model does not represent physical situation**. Must consider the model **without any diffusion**

- Flutter Analysis: typically based on fully linear models to determine onset conditions of the flutter. Rather than large displacement nonlinear effects responsible for post-flutter dynamics -LOC

Why rotational forces model is inappropriate for studying stabilization of flutter.?

FACT: Experimental finding: **dispersion [flow] "stabilizes" the structure.**

What does it say about modeling.?

This is impossible with rotational forces accounted for; i.e.Δu_{tt} added.

$$u_{tt} + \gamma \Delta u_{tt} + \Delta^2 u + f(u) = tr_U(\Phi)$$

HERE IS WHY.

Hidden stabilizing effect of the flow.

$$u_{tt} + \Delta^2 u + f(u) = \mathbf{tr_U \Phi}$$

becomes nonlinear PDE with a delay

$$u_{tt} + \Delta^2 u + f(u) = -\mathbf{tr_U(u)} + \mathbf{q(u, t, x, y)}$$

$$q(u,t,x,y) = \int_0^{t^*} ds \int_0^{2\pi} d\theta D^2(u(x-(U+sin\theta)s, y-scos\theta, t-s), DELAY$$

$$u_{tt} + \underbrace{u_t}_{stabilizes} + \Delta^2 u + f(u) = -\underbrace{Uu_x + q(u,t,x,y)}_{destabilizes}$$

Rotational model:

$$u_{tt} - \gamma \Delta u_{tt} + \Delta^2 u + f(u) = -\mathbf{tr_U \Phi} \text{ becomes}$$

$$u_{tt} - \gamma \Delta u_{tt} + \Delta^2 u + f(u) = -\mathbf{tr_U(u)} + \mathbf{q(u, t.x.y)}$$

$$u_{tt} - \gamma \Delta u_{tt} + \underbrace{u_t}_{does \;\; not \;\; stabilize} + \Delta^2 u + f(u) = \underbrace{Uu_x + q(u, t, x, y)}_{destabilizes}$$

$$u_{tt} - \gamma \Delta u_{tt} + \underbrace{u_t - \gamma \Delta u_t}_{stabilizes} + \Delta^2 u + f(u) = \underbrace{Uu_x + q(u, t, x, y)}_{destabilizes}$$

Conclusion: To stabilize structure : needs to add Δu_t . Flow does not harvest such term. Flow does not stabilize the rotational model. Rotational or diffusive model inappropriate for modeling flutter.

Conclusions

1. Structural model must be **irrotational and nonlinear** [large displacements] -confirming the experiment.
2. **Rotational model** yields smoother solutions by adding ONE DERIVATIVE making $f(u)$ compact. Simplifies the analysis **however not physical**.
3. The original model brings aboard **interesting mathematics:**
   - harmonic analysis, $f(u)$ becomes supercritical
   - non dissipative, lack of Lyapunov dynamical system theory: Uu_x
   - PDE dynamics with the delay: $q(u)$ and its destabilizing effects.
   - Lack of compactness, lack of regularity.

Gives rise to NEW Techniques

- PDE, harmonic and microlocal analysis, weak Hardy spaces,
- Dynamical Systems Theory for Non-Dissipative systems.
- Mixed-nonlocal boundary conditions.**Nonlocal analysis enters.**

Main Results- Overview for Neuman BC

- Existence and Hadamard wellposedness of **finite energy solutions** .
- Solutions are bounded for ALL TIMES. - CRITICAL role of **nonlinearity.** (**False for the linearization**)
- **Subsonic case:** with the feedback control damping acting on the panel subject to clamped BC all solutions stabilize to the **stationary states.**

Conclusion: Flutter can be eliminated

- **Supersonic and subsonic case:** With the flow data compactly supported all weak solutions of the structure **without any dissipation** converge to a **finite dimensional set** .

Conclusion: Undamped dynamics is finite dimensional asymptotically.

Previous analysis for regularized- parabolic like models.

Analysis of the original model: Critical role of

- Hyperbolicity-propagation of stability harvested from the flow.
- Nonlinearity at the critical/supercritical level. Compensated compactness/harmonic analysis.
- Sharp Trace Hyperbolic Theory -carriers of propagation
- New tools in dynamical systems and the theory of attractors.

Stability out of a Thin Air.

Model: Energies

- **Plate** $\mathcal{E}_{pl}(t) = \frac{1}{2}(||u_t||^2 + ||\Delta u||^2 + ||\Delta \mathcal{F}(u)||^2)$

- **Flow**: $\mathcal{E}_{fl}(t) = \frac{1}{2}(||\phi_t||^2 + ||\nabla \phi||^2 - U^2 ||\partial_x \phi||^2)$

- **Interactive**: $\mathcal{E}_{int}(t) = 2U < u_t, \gamma[\phi_x] >_{\partial D}$

$$\mathcal{E}_{pl}(t) + \mathcal{E}_{fl}(t) + \mathcal{E}_{int}(t) = Constant$$

Total Energy -why it is interesting?

- $\mathcal{E}(t) = \mathcal{E}_{pl}(t) + \mathcal{E}_{fl}(t) + \mathcal{E}_{int}(t)$.

Balance of Energy: $\mathcal{E}(t) = \mathcal{E}(s)$ **NO DISSIPATION**.

Hidden dissipation - dispersive effects to account for.

- $U < 1$, $\mathcal{E}_{fl} = \frac{1}{2}\left(||\phi_t||^2 + ||\nabla\phi||^2 - U^2||\partial_x\phi||^2\right) \sim ||\phi_t||^2 + ||\nabla\phi||^2$
- $U = 1$, $\mathcal{E}_{fl} \sim ||\phi_t||^2 + ||\partial_z\phi||^2 + ||\partial_y\phi||^2 + 0 \cdot ||\partial_x\phi||^2$
- $U > 1$, $\mathcal{E}_{fl} \sim ||\phi_t||^2 + ||\partial_z\phi||^2 + ||\partial_y\phi||^2 - (U-1)||\partial_x\phi||^2$

Nonlinear Semigroup -Generation

Neumann data for the Flow.

Theorem (J. Webster,NA 2011, Chueshov, I.L, Webster 2013 JDE)

- *Flow-structure interaction generates a continuous nonlinear semigroup*

$$S_t : H \to H = H_0^2(\Omega) \times L_2(\Omega) \times H^1(D) \times L_2(D)$$

- *Semigroup S_t is* **bounded** *for all $t > 0$ ($U < 1$).*
- *For compatible and suitably smooth initial data the corresponding solutions are* **smooth and global**.

Main Result - Subsonic and Supersonic

Theorem (Finite dimensional attracting set, CMPDE 2014)

- *Let $U \neq 1$, Consider plate solutions in $H_{pl} = H_0^2(\Omega) \times L_2(\Omega)$. Then, there exists a compact set $U \in H^3 \times H^2 \subset H_{pl}$ of finite fractal dimension such that*

$$lim_{t\to\infty} dist\{(u(t), u_t(t)), U\} =$$

$$lim_{t\to\infty} inf_{u_0,u_1 \in U}[||u(t) - u_0||^2_{2,\Omega} + ||u_t(t) - u_1||^2_{0,\Omega}] = 0$$

for all **compactly supported initial conditions** *corresponding to the flow.*

Strong Stability- Subsonic case

$\mathcal{N}$ denotes stationary solutions. Assume it is **finite.** (generically true. Can be eliminated :Haraux, Lojasiewicz lemma:)

Theorem (Stability, $U < 1$ and $k > k_0 > 0$, SIMA 2016)

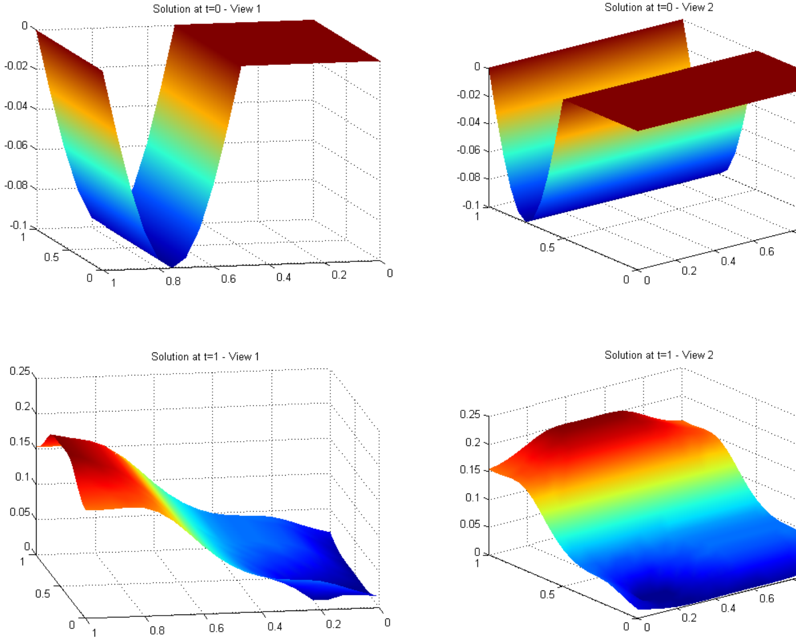
Let $U < 1$. Then any weak solution with **compactly supported flow initial data** *stabilizes to a stationary set. There exist $(u_0, u_1, \Phi_0, \Phi_1) \in \mathcal{N}$ such that for all $R > 0$.*

$$lim_{t\to\infty}||u(t) - u_0||^2_{2,\Omega} + ||u_t(t) - u_1||^2_{0,\Omega} = 0$$

$$lim_{t\to\infty}||\Phi(t) - \Phi_0||^2_{1,B(R)} + ||\Phi_t(t) - \Phi_1||^2_{0,B(R)} = 0$$

where $B(R)$ denotes a ball of radius R.

CONSEQUENCE: Flutter can be eliminated by applying damping to the structure only. Nonlinear effects are critical


Solution at t=0 - View 1
0
-0.02
-0.04
-0.06
-0.08
-0.1
1
0.5
0
1
0.8
0.6
0.4
0.2
0
Solution at t=0 - View 2
0
-0.02
-0.04
-0.06
-0.08
-0.1
1
0.5
0
0
0.2
0.4
0.6
Solution at t=1 - View 1
0.25
0.2
0.15
0.1
0.05
0
1
0.5
0
1
0.8
0.6
0.4
0.2
0
Solution at t=1 - View 2
0.25
0.2
0.15
0.1
0.05
0
1
0.5
0
0
0.2
0.4
0.6

Lyapunov stability with a damping

4

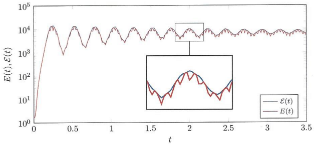


FIGURE 5. Plot of energies for $k = 1$ and $U = 2U_{\text{crit}}$, nonlinear model. fig:f5

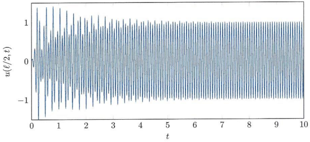


FIGURE 6. Plot of u at beam midpoint for $k = 1$ and $U = 2U_{\text{crit}}$, nonlinear model. fig:f11

Convergence to an equilibrium

0.3. **Influence of the b_0 Parameter.** The stabilization term $-b_0\|u_x\|^2 u_{xx}$ in the Berger dynamics limits the . In Figure 7 the midpoint displacement for $U = 600$, $k = 1$, and $b = 0$ is given for several values of b_0. There is little difference in the dynamics for $b_0 = 0.001$ and $b_0 = 1$, however for $b_0 = 100$ note that both frequency and amplitude of the oscillations are somewhat decreased. The difference in energy $E(t)$ is more stark for the largest b_0, as shown in Figure 8.

need more here

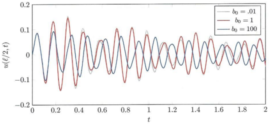


FIGURE 7. Plot of u at beam midpoint for $U = 600$, $k = 1$, $b = 0$, and varying b_0, nonlinear model.

fig:f17

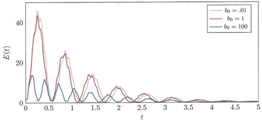


FIGURE 8. Plot of $E(t)$ for for $U = 600$, $b = 0$, $k = 1$, and varying b_0, nonlinear model.

fig:f18

convergence to an equilibrium

0.4. **Influence of the b Parameter.** The parameter b in the Berger dynamics translates to a physical in-plane (in-axis in 1-D) tension if $b < 0$ and compression if $b > 0$. To demonstrate the effect of this parameter on the beam dynamics, the beam midpoint displacement is plotted in Figure 9 for $U = 600$, $k = 1$, $b_0 = 0.1$, and several choices of b. Note that in-plane compression has a significant effect on the dynamics, while in-plane tension acts as a smoothing term on the beam oscillations and reduces the frequency of oscillations.

reference earlier in the paper

say how the term contributes to the structure of the stationary set

fig:f10

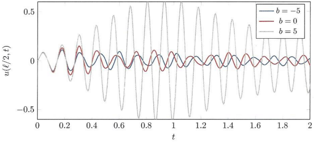


FIGURE 9. Plot of u at beam midpoint for $U = 600$, $k = 1$, $b_0 = 0.1$, and varying b, nonlinear model.

fig:f10

Convergence to a non-trivial steady state

0.5. **Convergence to a Non-trivial Steady State.** For certain parameter combinations, the presence of excess in-plane compression leads to a "buckled" plate/beam configuration. For $b_0 = 1$, the parameter $b = 50$ is large enough for $U = 100$ to impart this behavior and a plot of the midpoint beam displacement is given in Figure fig:f12 10. Note that the transient behavior decays more rapidly to the nontrivial steady state for larger values of k. A plot of the energy $E(t)$ for different values of k is given in Figure fig:f13 11. Regardless of the choice of k, all simulations for $U = 100$, $b_0 = 1$, and $b = 50$ converge to the same nontrivial steady state and same linear energy. A plot of the nontrivial steady state u (the buckled beam displacement) is given in Figure fig:f14 12.

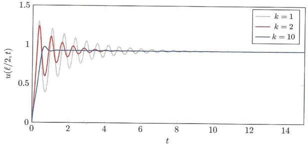


FIGURE 10. Plot of u at beam midpoint for $U = 100$, $b = 50$, $b_0 = 1$, and varying k, nonlinear model.

fig:f12

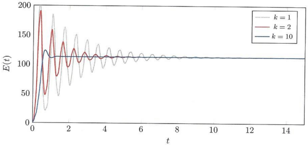


FIGURE 11. Plot of $E(t)$ for for $U = 100$, $b = 50$, $b_0 = 1$, and varying k, nonlinear model.

fig:f13

Buckling and Convergence to two different non-trivial steady states

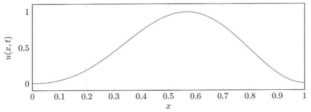


FIGURE 12. Plot of steady-state beam displacement for $U = 100$, $b = 50$, $b_0 = 1$, and varying k, nonlinear model. fig:f14 .

It is also possible to observe different choices of k to require convergence to different steady states. For $U = 100$, $b_0 = 1$, and $b = 100$, the choices $k = 1$ and $k = 2$ actually produce nontrivial steady states that are negatives of each other. In Figure 13 (fig:f15) the midpoint displacement is plotted for these two cases, and the energies are give in Figure 14 (fig:f16).

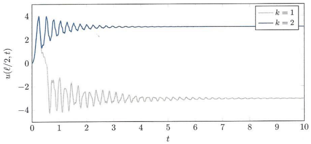


FIGURE 13. Plot of u at beam midpoint for $U = 100$, $b = 100$, $b_0 = 1$, and varying k, nonlinear model. fig:f15

Convergence to a limit cycle

0.6. **Convergence to a Limit Cycle.** In Figure fig:f6 15, computed energies (log-scale) are plotted for a large flow velocity ($U = 5000$), significant in-axis compression parameter ($b = 50$), and $b_0 = 1$ for selected values of the damping parameter k. The sensitivity of the dynamics to the damping parameter can be seen by noting the relative rate of initial decay of energy as k increases. Plots of the beam midpoint displacement are given in Figure fig:f7 16 - note the quick decay to the oscillatory limit cycle for larger values of k.

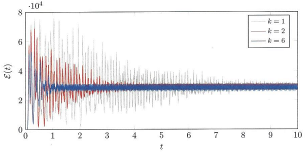


FIGURE 15. Plot of $\mathcal{E}(t)$ for for $U = 5000$, $b = 50$, $b_0 = 1$, nonlinear model. fig:f6

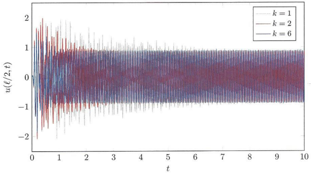


FIGURE 16. Plot of u at beam midpoint for $U = 5000$, $b = 20$, $b_0 = 1$, nonlinear model, varying k. fig:f7

It is possible to induce several different phenomena by manipulating parameter values. In Figure fig:f8 17 the midpoint displacement is shown for $U = 5000$, $k = 100$,

Convergence to a limit cycle. Flutter

$b = 5000$, and $b_0 = 1000$. Note the initial transients are damped out quickly and the dynamics converges to a limit cycle.

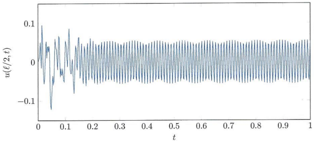


FIGURE 17. Plot of u at beam midpoint for $U = 5000$, $b = 5000$, $b_0 = 5000$, $k = 101$ nonlinear model.

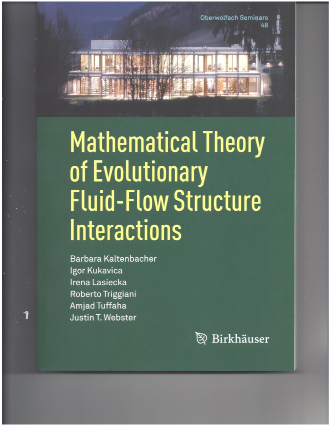
fig:f8

PROOFS given in

THM 1: Wellposedness of Energy Solutions $U \in [0,1) \cup (1,\infty)$
JDE 2013-I. Chueshov,IL, J.Webster
THM 2: Attracting Sets for the Structure $U \in [0,1) \cup (1,\infty)$
CMPDE 2014-I. Chueshov,IL,J. Webster
THM 3: Strong Stability for the Flow-Structure $U \in [0,1)$
SIMA 2016- I.L. J.Webster

Review Paper: Mathematical Theory of Flow Structure Interaction
AMO 2016 -I.Chueshov, E. Dowell, I.L. J. Webster.
Oberwolfach Seminars: Flow-plate interactions: stabilization and control. Oberwolfach Semin., 48, Birkhauser-Springer, 2018. I.L. and J. Webster

Oberwolfach Seminars -Springer-Birkhauser, 2018



Challenges-Why it is interesting.

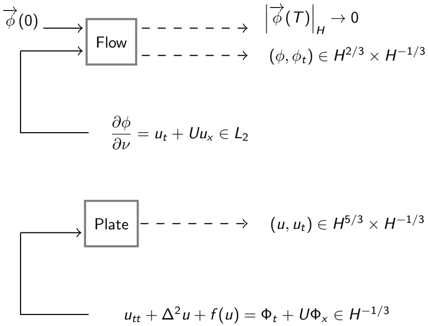
Mathematical Reasons:

- Supercritical nonlinearity.
- A "natural" decoupling of PDE components does not work?
- "Classical dynamical system methodology " fails. New "non dissipative" theory developed in I.Chueshov, I.L. Springer's Monograph 2010.

Working with the correct PDE model [**nonlinear, irrotational and non-diffusive**] solves Flutter problem

Natural PDE Splitting : Good Idea which Does not Work

"Loosing" derivatives: R. Triggiani, D. Tataru : 1993-1999



Loss of $\frac{1}{3}$ derivative

Lopatiniski condition violated

Hyperbolic Neumann map

Why are we "loosing" derivatives?

$$\Phi_{tt} = \Delta\Phi \quad \text{in } D \times (0,T), \quad \Phi(0) = \Phi_t(0) = 0$$

$$\partial_\nu \Phi = g \quad \text{on } \partial D \times (0,T)$$

$$g \to (\Phi, \Phi_t) \quad \text{Hyperbolic Neumann map } N_h$$

$$N_h g \equiv (\Phi, \Phi_t), \quad \textit{Lopatinski} \quad \textit{fails}$$

$$N_h : L_2(L_2(\partial D)) \to C(H^{2/3}(D) \times H^{-1/3}(D))$$

Loss of $1/3$ derivative when dimension of D > 1 .

Consequences

When $u_t \in H^1(\Omega)$ [regularized models] then $g = u_t + Uu_x \in H^1$ and

$$Ng \in K - compact \subset H^1 \times L_2$$

When $u_t \in L_2(\Omega)$ then $g \in L_2$ and

$$Ng \in H^{2/3}(D) \times H^{-1/3}(D)$$

Loss of $1/3$ derivative when dimension of D > 1

Wellposedness : Energy function

$$E(t) = E_u(t) + E_\Phi(t) + E_{int}(t)$$

$$E_u(t) = \int_\Gamma [|u_t|^2 + |\Delta u|^2 + |\Delta F(u)|^2] d\Gamma$$

$$E_\Phi(t) = \int_\Omega [|\Phi_t|^2 + \int_\Omega [|\nabla \Phi|^2 - \mathbf{U}|\mathbf{\Phi_x}|^2] d\Omega$$

$$E_{int}(t) = \mathbf{U} \int_\Gamma \Phi u_x d\Gamma$$

Energy balance : $E(t) = E(s)$ for all s, t

- E_Φ may **degenerate** when $U > 1$ (supersonic)

$$\Delta - \mathbf{U} D_x^2 = (1 - \mathbf{U}) D_x^2 + D_y^2 + D_z^2$$

- $E(t)$ is not necessarly positive. (E_{int} **-indefinite**).

Properties of the Energy

- $E_\Phi(t)$ with $U > 1$ is non-positive -"hyperbolic type"
- $E_{int}(t)$ has indefinite sign , however we have energy balance

$$E_\Phi(t) + E_{pl}(t) + E_{int}(t) = Constant, t \in R$$

Bad Energy, Good Energy Balance

Supersonic energy

$$E(t) = E_u(t) + E_\Phi(t) + E_{int}(t)$$

$$E_u(t) = \int_\Omega [|u_t|^2 + |\Delta u|^2 + |\Delta F(u)|^2], \quad E_{int}(t) = U \int_\Omega \Phi_x(t) u(t)$$

$$E_\Phi(t) = \int_\Omega [|\Phi_t(t) + U \frac{\partial}{\partial x} \Phi(t)|^2 + |\nabla \Phi(t)|^2] d\Omega$$

Energy relation : $E(t) = E(s) - \mathbf{U} \int_s^t \int_\Gamma (u_t \Phi_x|_\Gamma + U u_x \Phi_x|_\Gamma) dx ds$

Good Energy, Bad Energy Balance

Properties of supersonic energy

- Dissipation integral of indefinite sign and not defined for finite energy solutions -

$$< u_t, \Phi_x >_{\partial D}$$

$\Phi_x|_{\partial D}$ **not defined in** $L_2(\partial D)$ for $\Phi \in H^1(D)$

$$< u_t, \Phi_x >_{\partial D} \quad \sim L_2 \cdot H^{-1/2}????$$

Loss of dissipativity and loss of regularity. NOT A GOOD SPELL.

Plate-Structure. Nonlinearity becomes supercritical ($u_t \in L_2$)

Flow- Interface traces not defined on the energy space .

Harmonic Analysis and Microlocal Analysis Enter the Game.

1. **to deal with super linearity of the plate motion** and
2. **to propagate stability harvested by the sheer flow**

Prove Two Regularity Results : (1) for the flow and (2) for the structure

For the flow

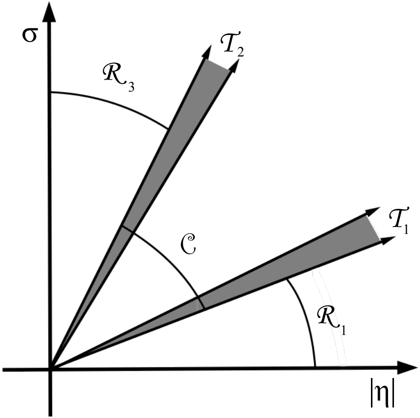
Lemma (Trace Regularity → compensated compactness)

(Microlocal)

$$\Phi \in H^1(Q), \;\; and \;\; \partial_\nu \Phi \in L_2(\Sigma) \Rightarrow \Phi_t|_\Gamma \in L_2(0, T; H^{-1/2}(\Gamma))$$

$\Phi_t \in L_2(\Omega)$ does not allow for application of the trace operator.

New trace estimates for aeroelastic dynamics to be discovered.



In $\mathcal{R}_3$-hyperbolic - L_2 regularity, In $\mathcal{R}_1$-elliptic - L_2 regularity
In $\mathcal{C}$-characteristic - $H^{-1/2}$ regularity -loss of regularity.

$$u_{tt} + \Delta u = [\mathcal{F}(u), u], \quad \mathcal{F}(u) = \Delta^{-2}[u, u]$$

$$[u, v] : H^2 \times H^2 \to L_1 \subset H^{-\epsilon} \Rightarrow$$

$$\mathcal{F}(u) : H^2 \times H^2 \to H^{3-\epsilon} \Rightarrow$$

$$[\mathcal{F}(u), u] : H^2 \times H^2 \to H^{-\epsilon}, \epsilon > 0$$

Theorem (I.L. D.Tataru- Airy's stress function)

$$[u, v] : H^2 \times H^2 \to H_1[Hardy = F^{1,0}] \Rightarrow$$

$$\mathcal{F}(u) : H^2 \times H^2 \to W^{2,\infty} \Rightarrow$$

$$[\mathcal{F}(u), u] : H^2 \times H^2 \to L_2$$

There is **no loss** of ϵ. harmonic analysis+compensated compactness

Stability and reduction to finite dimensional model: STRATEGY

1. **STEP 1:** $k > 0$. Strong convergence to the set $\mathcal{N}$ of orbits driven by **smooth structural** data
   - Use **dispersion** estimates for the flow driven by the initial conditions
   - Analysis of the coupling via Neumann map: "tour de force " loosing derivatives . **Strong stability for smooth** initial data. PDE decoupling ($k > 0$) .

$$S_t(Y_r) \to \mathcal{N}, Y_r \in D(A)$$

2. **STEP 2:** $k > 0$ Uniform Hadamard sensitivity uniformly in $t > 0$ when $||Y_r - Y|| \leq \epsilon$.

$$||S_t(Y_r) - S_t(Y)|| \leq \epsilon c(\int_0^t ||u_t||)$$

Controlling the **rate of attraction** $||u_t|| \in L_1$?. We only know $||u_t|| \in L_2$.

- **STEP 3 :** $k \geq 0$ Back to the structure. Construct **attractor** $\mathcal{A}$ for the structure only.
  Big Gun. **No dissipation, no compactness**. But "hidden" dissipation harvested from the flow and "hidden" compactness.
  - $U(t)(B) \to \mathcal{A}$. Prove **smoothness** on that attractor $\mathcal{A}$.
  - **Tool**: **Quasistability estimate**. Use **backward invariance**.

$$|S_T^U(u) - S_T^U(v)|_H \leq 1/10|u - v|_H + C_T sup_{0,T}|S_t^U u - S_t^U v|_{H_1}$$

for $u, v \in \mathcal{A}$, $H \subset H_1$ compact embedding. $H_1 \sim [D(A^{\epsilon})]'$

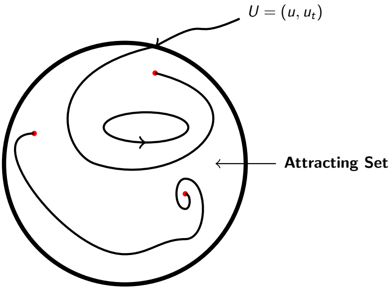
- **STEP 4:**
  - either $U(t)$ enters $\mathcal{A}$ -OK since smooth -go to Step 1.
  - or approaches $\mathcal{A}$. Question: **at which rate?**

- **STEP 5,** $k \geq 0$: Prove an existence of exponential attractor $\mathcal{A}_e \supset \mathcal{A}$.

$$dist(U(t)B, \mathcal{A}_e) \leq ce^{-\omega t}$$

  The rate is OK, but **smoothness???**. Difficulty: $\mathcal{A}_e$ is only positively invariant.
- **STEP 6,** $k \geq 0$; Prove smoothness of the exponential attractor $\mathcal{A}_e$. Using **quasi stability estimate** for a suitable decomposition of the flow which filters out initial data (Zelik, Vishik). **Attraction at the L_1 rate to a smooth set.** Go back to Step 1.

BIG GUN - for Step 3.
Uniform Convergence with respect to $Y_0 \in B_H$ for the structure only.
Hidden compactness of the delay term $q(u)$ and **hidden** dissipation due to the flow : the term u_t appears "out of the blue".
No dissipation on the flow. Study of systems with delay via Quasistability estimate. The attracting set is smooth, possibly chaotic.



$k = 0$. Hidden stabilizing effect of the flow

$$u_{tt} + \Delta^2 u = [\mathcal{F}(u), u] + p(u,t,x,y)$$

$$p(u,t,x,y) \equiv -(\mathbf{u_t} + \mathbf{U u_x}) + q(u,t,x,y)$$

$$u_{tt} + u_t + \Delta^2 u = [\mathcal{F}(u), u] + U u_x + q(u,t,x,y)$$

$$q(u,t,x,y) = \int_0^{t^*} ds \int_0^{2\pi} d\theta D^2(u(x-(U+sin\theta)s, y - scos\theta, t-s)$$

$$D^1 = e^{-i\theta} \cdot \nabla^{\perp}_{x,y} = sin\theta \frac{\partial}{\partial x} + cos\theta \frac{\partial}{\partial y}$$

$$t^* = inf\{t, \vec{x}(U,\theta,s) \notin \Gamma, \vec{x} \in \Gamma\}, \vec{x} \equiv (x - s(U+sin\theta), y - scos\theta)$$

Attractor for the plate, $U \neq 1$

Let $k \geq 0$**, no damping** **Big Gun** leads to

Theorem (Chueshov, I.L. Webster CMPDE 2014)

- *Let* $U \neq 1$*. Consider plate solutions in* $H_{pl} = H_0^2(\Omega) \times L_2(\Omega)$*. Then, there exists a compact set* $U \in H_{pl}$ *of finite fractal dimension such that*

$$lim_{t\to\infty} dist\{(u(t), u_t(t)), U\} =$$

$$li_{t\to\infty} inf_{u_0, u_1 \in U}[||u(t) - u_0||^2_{2,\Omega} + ||u_t(t) - u_1||^2_{0,\Omega}] = 0$$

  for all **compactly supported initial conditions** *corresponding to the flow.*
- *There exists* **compact** *"attractor" for the plate .*

The analysis reduced to a finite dimensional invariant set
Determination of the fluttter speed

Uniform Hadamard for the full system

$$Y = (\Phi, \Phi_t, u, u_t)$$

$$|Y(t) - Y_m(t)^2|_H \leq C^{|d|_{L_1}} |Y(0) - Y_m(0)|_H^2$$
$$d(t) = |u_t(t)|_{Y_{pl}} + |u_{m,t}(t)|_{Y_{pl}}$$

We know only that $|u_t|_{L_2(0,\infty)} + |u_{m,t}|_{L_2(0,t\infty} < \infty$

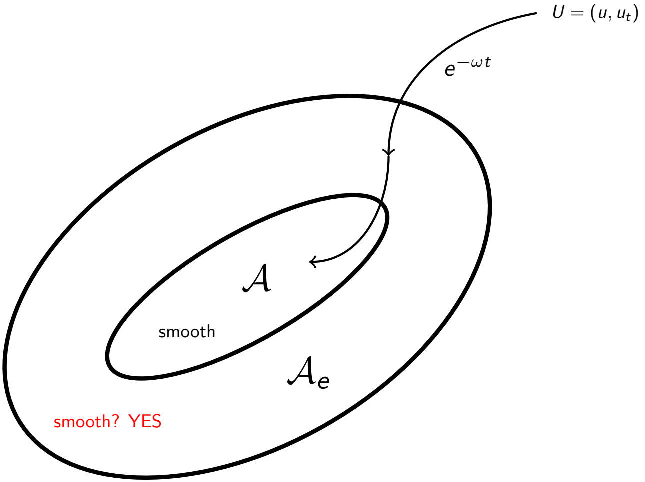
We need $d \in L_1(0,\infty)$ rather than $d \in L_2(0,\infty)$.

Dychotomy Principle

The trajectory $U = (u, u_t)$ EITHER enters the attractor
OR approaches the attractor with $L_1(0, \infty)$ rate.

In the first scenario - previously developed "smooth analysis" applies.
In the second scenario: approximate the trajectory by smooth and L_1 convergent solutions. Leads to **exponential attractors**.

- Exponential attractor $\mathcal{A} \subset \mathcal{A}_e$: convergence to the attractor is exponential. **No information on the smoothness**.
- Regular attractor $\mathcal{A}$: Smooth but **no information on the rate** of convergence.
- A. Miranville and S. Zelik: Survey article in the Handbook on DE -2010. Closing this gap (even for discrete dynamical systems) is in general **open problem**.
- **Finally -Exponential attractor $\mathcal{A}_e$ for the structure is SMOOTH.**



$$\text{dist}(U(t), \mathcal{A}_e) \leq Ce^{-\omega t}$$

Back to the flow

- Flow provides **"hidden" dissipation**
- Structure - "plate" provides **"hidden" asymptotic regularity on the attracting set**.
- **Propagating** these properties through the entire system -main challenge of the problem.

SUBSONIC FLOW:

Flutter can be eliminated by applying damping to the structure.

SUPERSONIC FLOW:

Flow **has stabilizing effect**.With **no damping** on the structure solutions are driven to a finite dimensional set. PDE dynamics reduced to ODE dynamics. Structure of the set : **chaotic, periodic orbits, limit cycles.**
Finite dimensional Boundary Control Theory: LQG, H-J theory.

Important Message

Stabilizing effect of the flow exhibited only for a correct model -

- nonlinear,
- without rotational inertia,
- without diffusive effects.

K-J boundary conditions: zero pressure off the wing and free-clamped on the structure

A nonlinear plate and perturbed wave, coupled at the interface $\Omega \subset \mathbb{R}^2$:

$$\begin{cases} u_{tt} + \Delta^2 u + \mathbf{f}(\mathbf{u}) = p_0 + tr_U(\phi)|_\Omega \ \ in\ \Omega \\ BC(u) = Clamped\ on\ \partial\Omega \\ (\partial_t + U\partial_x)^2\phi = \Delta\phi\ \ in\ R^3_+ \end{cases}$$

$$\partial_\nu \phi\big|_{z=0} = -tr_U(u) \ \ on\ \ \Omega,\ \phi_t + U\phi_x = 0 \quad outside\ \Omega$$

$f(u) = -[\mathcal{F}(u), u]$, $tr(u) \equiv u_t + Uu_x$

Work in progress

Further Directions.

- **TRANSONIC** CASE. [$U = 1$]. Numerical evidence of shocks . Analysis **must account for nonlinearity of the flow**.
- **Kutta Jukovsky** boundary conditions and $U \geq 1$.

$$\phi_t + U\phi_x = 0 \quad off \quad the \quad wing.$$

Mathematical interest: **invertibility of finite Hilbert transforms.** L_p **theory for** $p \neq 2$**.**
Chueshov, I.L, Webster - DCDS 2014.

- **Free -clamped** boundary conditions on the plate. Experiments indicate hysteresis -not predicted by the present model. Use **NSE** to model the flow.
- **Nonlinear Flow equation**: NS or Euler

References

- I.Lasiecka, B. Priyasad, R. Triggiani. Finite dimensional boundary control of 3 D fluids. **2018**.
- I. Chueshov and I. Lasiecka, *Long Time Behavior of Second Order Evolutions with Nonlinear Damping* , **Memoires, AMS,** 2008
- I. Chueshov and I. Lasiecka, *Von Karman Evolutions, Wellposedness and Long time Behavior* , **Monographs, Springer Verlag**, 2010.
- E. H. Dowell *Aeorelasticity of plates and shells* , Nordhof 2004.
- I. Lasiecka, R. Triggiani , *Control Theory of PDE's*, **Cambridge University Press, 2000**
- M.A, Horn, I. Lasiecka , D. Tataru . Global existence, uniqueness of solutions to V. Karman equations **Diff. Int. Eq**, 1996

- M.Ignatova, I. Kukavica, I.Lasiecka, A.Tuffaha , Global solutions in a 3-D fluid structure interactions, **Nonlinearity, 2014, 2017**
- I. Chueshov, I. Lasiecka , J. Webster: Evolutions in Supersonic flow-structure interactions, **JDE 2013.**
- I. Chueshov, I. Lasiecka, J. Webster: Long time dynamics of delayed plates. **Communications on PDE, 2014.**
- I. Chueshov, I. Lasiecka, How to eliminate flutter, **SIAM Mathematical Analysis, 2016**
- I. Chueshov , E. Dowell. I. Lasiecka and J. Webster, Mathematical Aeroelasticity-Survey, **AMO, 2016**
- B. Kaltenbacher, I.Kukavica, I Lasiecka, R. Trigiani, Mathematical theory of Fluid Structure interactions, **Birkhauser , 2017**
- F. Gazzola, Torsional instability in suspension bridges: the Tacoma Narrows Bridge case.**Comm. Nonlinear Sci. Numer. Simul. 2017**
- I. Chueshov, Dynamics of quasi-stable dissipative systems. **Universitext. Springer, Cham, 2015.**

- E. Dowell, A Modern Course in Aeroelasticity,*Springer*, **2014**
- E. Dowell, K. McHugh, Equations of motion for an inextensible beam under large deflections,**Journal of Fluids and Structures, 2016.**
- M.M. Kasem and E.H. Dowell, A study of the natural modes of vibrations and aeroelastic stability of a plate with a piezoelectric material, **Smart Materials and Structures, 2018.**
- M.R. Saga and E.H. Dowell, Nonlinear Structural, Inertial and Damping Effects in an Oscilating Cantilever Beam, **Nonlinear Dynamics, vol 1, Springer, 2019.**
- E.H. Dowell and D. Tang, Experimental Aeroelastic Models designs and wind tunnel testing for correlation with new theory, **Aerospace, 2016.**
- E. Dowell and D. Tang, Aeroelastic response and energy harvesting from a cantilevered piezoelectric laminated plate, **Journal of Fluids and Structures, 2018.**