

Departamento de Matemáticas

Facultad de Ciencias Naturales

Recinto de Río Piedras

**MATE
3152**

Apellidos: _____ Nombre: _____

No. de estudiante: _____ Profesor: V. Keyantuo

Examen Final 24 de Mayo de 2007 # de sección: 003

Para obtener crédito muestre todo su trabajo. Explique claramente su contestación.

Note. All answers should be justified.

(1) (30 pts) Evaluate the following limits

(a) Find $\lim_{x \rightarrow \infty} \frac{5x + 1}{4x}$

(b) Find $\lim_{x \rightarrow \infty} \frac{1}{\sqrt{x}}$

(c) Find $\lim_{x \rightarrow 0} \frac{\sqrt{4-x} - 2}{x}$

(d) Find $\lim_{x \rightarrow \infty} \frac{\ln(x+2)}{\sqrt{x+2}}$

(e) Find $\lim_{x \rightarrow \infty} \frac{e^x}{4^x + 3}$

(f) Find $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$

(2) (10 pts)

(a) Use the ratio test to prove that the series $\sum_{n=0}^{\infty} \frac{2^n}{n!}$ is convergent.

(b) Find $\lim_{n \rightarrow \infty} \frac{2^n}{n!}$

(3) (25 pts) Determine if the following sequences are convergent and find the limits:

(a) $\lim_{n \rightarrow \infty} \frac{5n + 2}{4n}$

(b) $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}}$

$$(c) \lim_{n \rightarrow \infty} n \left(\sqrt{4 - \frac{1}{n}} - 2 \right)$$

$$(d) \lim_{n \rightarrow \infty} \frac{\ln(n+2)}{\sqrt{n+2}}$$

$$(e) \lim_{n \rightarrow \infty} \frac{e^n}{4^n + 3}$$

(4) (25 pts) Evaluate each of the limits. Justify your answers.

$$(a) \lim_{x \rightarrow \infty} \frac{\frac{x^3}{(1+x^3)^2}}{\frac{1}{x^3}}$$

$$(b) \lim_{x \rightarrow \infty} \frac{\sin^2 \frac{1}{\sqrt{x}}}{\frac{1}{x}}$$

$$(c) \lim_{x \rightarrow \infty} \cos\left(\frac{1}{x^2}\right)$$

(d) $\lim_{x \rightarrow \infty} \frac{\sin \frac{1}{x^2}}{\frac{1}{x^2}}$

(e) $\lim_{x \rightarrow \infty} \frac{\frac{1}{\sqrt{2+x}}}{\frac{1}{\sqrt{x}}}$

(5) (15 pts)

(a) Find $\lim_{x \rightarrow \infty} \frac{x^{3/2}}{x^{5/2} + 2x - 1}$

(b) Show that the function $f(x) = \frac{x^{3/2}}{x^{5/2} + 2x - 1}$ decreases on $[10, \infty)$.

(c) Show that the series $\sum_{n=1}^{\infty} (-1)^n \frac{n^{3/2}}{n^{5/2} + 2n - 1}$ is convergent.

- (6) (28 pts) For each of the following series, examine whether it converges or not. Justify your answers.

(a) $\sum_{n=1}^{\infty} \frac{n^3}{(1+n^3)^2}$

(b) $\sum_{n=1}^{\infty} \sin^2 \frac{1}{\sqrt{n}}$

(c) $\sum_{n=1}^{\infty} \cos \frac{1}{n^2}$

(d) $\sum_{n=1}^{\infty} \sin \frac{1}{n^2}$

(e) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{2+n}}$

(7) (25 pts) Compute the following indefinite integrals.

(a) $\int \frac{x^2}{\sqrt{4-9x^2}} dx$

(b) $\int \frac{\tan(3 \ln x)}{x} dx$

(c) $\int \tan^4 x dx$

(d) $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$

(e) $\int \frac{x}{1+x^4} dx$

(8) (10 pts) Obtain the partial fraction decomposition of

(a) $f(x) = \frac{x^2}{1+x}$

(b) $g(x) = \frac{x}{x^2-1}$

(9) (16 pts) Evaluate the definite integrals

(a) $\int_0^{\pi/2} \cos^3 x dx$

(b) $\int_0^1 \frac{1}{1 + \sqrt[3]{x}} dx$

(10) (15 puntos) Encuentre cada una de las siguientes derivadas.

(a) $\frac{d}{dx} [\sin^{-1}(e^x)] =$

(b) $\frac{d}{dx}(x^{e^x})$

(c) $\frac{d}{dx} \ln(x^3 + 3^x)$

(11) (10 puntos) Evaluar la integral indefinida $\int x^2 \cos x dx$.