Departamento de Matemáticas

Facultad de Ciencias Naturales
Recinto de Río Piedras

MATE 3152

Apellidos: ________________ ________________ Nombre: ________________
No. de estudiante: ____________________ Profesor: ____________________
Practica. Abril de 2007 # de sección: ________________

Para obtener crédito muestre todo su trabajo. Explique claramente su contestación.

(1) *(puntos)*
   (a) Find the focus and directrix of the parabola: $y^2 = 12x$.

   (b) Equations for normal and tangent lines to the parabola: $y^2 + 2y - 2x = 4$ at the point $A(2, 2)$.

   (c) Sketch the graph of the hyperbola: $10x^2 - 25y^2 = 100$.

   (d) Equations for tangent and normal lines to the ellipse: $\frac{x^2}{27} + \frac{y^2}{16} = 1$ at $A(3, \frac{4\sqrt{6}}{3})$.

   (e) Find the area of the surface generated by revolving the curve $x = 2 + \cos t$, $y = \sin t$ around the $x-$axis for $0 \le t \le 2\pi$.

(2) Examine whether the following improper integrals converge or not.
   (a) $\displaystyle\int_0^\infty \frac{x}{(16 + x^2)^2} dx$

   (b) $\displaystyle\int_{1/3}^3 \frac{dx}{\sqrt[3]{3x - 1}} dx$

   (c) $\displaystyle\int_{-1}^1 \frac{\sin^{-1} x}{\sqrt{1 - x^2}} dx$

   (d) $\displaystyle\int_1^4 \frac{dx}{x^2 - 4} dx$

(e) $\displaystyle\int_0^{\pi/2} \frac{\cos x}{\sqrt{\sin x}} dx$

(f) $\displaystyle\int_0^{\infty} \cos^2 x dx$

(g) $\displaystyle\int_0^{\infty} \sin(x^2) dx$

(h) $\displaystyle\int_{-\infty}^{\infty} \sin x dx$

(i) $\displaystyle\int_0^{\infty} \frac{1}{\sqrt{2+\cos x}}\, dx$

(3) *(puntos)* Encuentre cada una de los siguientes limites si existen.

(a) $\displaystyle\lim_{x\to 0} \frac{x^3}{4^x - 1} =$

(b) $\displaystyle\lim_{x\to 2+} \frac{\sqrt{2x}-2}{\sqrt{x-2}} =$

(c) $\displaystyle\lim_{x\to 0} \frac{\sin(2x) + ax + bx^3}{x^3}$
(discuss according to the values of a and b)

(d) $\displaystyle\lim_{x\to 0} \frac{a^x - (a+1)^x}{7x}$

(e) $\displaystyle\lim_{x\to 0^+} \frac{(\ln x)^\alpha}{x} = 0$ for every $\alpha \in \mathbb{R}$

(f) $\displaystyle\lim_{x\to 0} (x^2+1)^{\sinh x}$

(g) $\displaystyle\lim_{x\to \infty} (x^2+a^2)^{(1/x)^2}$

(h) $\displaystyle\lim_{x\to 0} [\frac{1}{\sin^2 x} - \frac{1}{x^2}] =$

(i) $\lim_{x\to\infty} [\frac{1}{x}\int_0^x \sin\frac{1}{t+1} dt] =$

(j) $\lim_{x\to\infty} \left(\frac{a^{1/x}+b^{1/x}}{2}\right)^x =$
(where a and b are positive).

(4) Find the area of the surface of revolution obtained by rotating the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ about the $y-$axis.

(5) Find the area of the surface of revolution obtained by rotating the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ about the $x-$axis.

(6) Let $\mathcal{P}$ be a parabola with horizontal axis, and focus F. Let M be a point on the parabola. Show that the angle between the tangent line to the parabola (at M) and the horizontal line through M is equal to the angle between the tangent line and the line (FM).

(7) Let $\mathcal{E}$ be an ellipse with foci F_1 and F_2. Let M be a point on the ellipse. Show that the lines F_1M and F_2M make equal angle with the tangent line to the parabola at M.

(8) Let $\mathcal{H}$ be a hyperbola with foci F_1 and F_2. Show that if M is a point on $\mathcal{H}$, then the angle between the tangent line at M and lines F_1M and F_2M are equal.

(9) Find the angle that eliminates the xy terms in the equation of the conic section.
(a) $xy = 1$

(b) $xy - y + x - 1 = 0$

(c) $2x^2 + 4\sqrt{3}xy + 6y^2 + (8-\sqrt{3})x + (8\sqrt{3}+1)y + 8 = 0$

(d) $3x^2 + 2\sqrt{3}xy + y^2 - 2x + 2\sqrt{3}y = 0.$