

Universidad de Puerto Rico  
Departamento de Matemáticas  
MATE 3018 – Final Exam – December 11, 2008

Apellidos: \_\_\_\_\_ Nombre \_\_\_\_\_  
No. Estudiante: \_\_\_\_\_ Profesor: Warma Sección OU1

(1) For each question **explain** your answer.

(a) (**2 Pts**)  $\log_3 \left( \frac{1}{81} \right) =$

(b) (**2 Pts**) Write in the form  $a + bi$ .

$$\frac{5i - 4}{3 - 4i} =$$

(2) Consider the function  $f(x) = 3x^2 + 5x - 15$ . Evaluate:

(a) (**2 Pts**)  $f(-1) =$

(b) (**3 Pts**)  $\frac{f(x) - f(-1)}{x + 1} =$

(c) (**3 Pts**)  $\frac{f(x + h) - f(x)}{h} =$

(3) Given that  $t_5 = 12$  and  $t_{24} = 88$  are terms of an arithmetic sequence, find:

(a) (**3 Pts**)  $t_{12} =$

(b) (**3 Pts**)  $S_{40} := \sum_{k=1}^{40} t_k =$

(4) Solve the following equations or inequations over the set of real numbers.

(a) **(3 Pts)**  $|-x + 7| \geq 10$

(b) **(3 Pts)**  $[\ln(x)]^2 - 6\ln(x) + 8 = 0$

(c) **(3 Pts)**  $\frac{3 + 5x}{x - 3} \geq 0$

(d) **(3 Pts)**  $x^2 - 4x + 3 \leq 0$

(5) Find the domain of each of the following functions:

(a) **(3 Pts)**  $h(x) = \frac{x + 2}{\sqrt{16 - x^2}}$

(b) **(3 Pts)**  $g(x) = \frac{x - 2}{x^3 - 8}$

(6) (6 Pts) Let  $P(x) = x^4 - 3x^3 + 5x^2 - x - 10$ . Verify that  $P(1 - 2i) = 0$  and find **all roots** of  $P(x) = 0$ .

(7) (3 Pts) Find the equation of the line that is perpendicular to the line  $2x - 6y = 15$  and passes through the point  $(-3, 4)$ .

(8) (4 Pts) Find the coordinates of the center and the radius of the circle with equation  $x^2 + y^2 + 5x + 7y = 0$ .

(9) (12 Pts) Find the exact value of:

(a) $\tan\left(\frac{11\pi}{12}\right) =$	(d) $\tan(75^\circ) =$
(b) $\tan\left(\frac{13\pi}{12}\right) =$	(e) $\cos(20^\circ)\cos(40^\circ) - \sin(20^\circ)\sin(40^\circ) =$
(c) $\tan\left(\frac{5\pi}{8}\right) =$	(f) $\sin(35^\circ)\cos(15^\circ) + \cos(35^\circ)\sin(15^\circ) =$

(10) Solve the following equations for  $0 \leq x < 2\pi$ .

(a) (**3 Pts**)  $4 \sin(x) \cos(x) = \sqrt{3}$

(b) (**3 Pts**)  $\sin(x) \cos(2x) + \cos(x) \sin(2x) = \frac{1}{2}$

(c) (**3 Pts**)  $\cos(x) \cos(3x) - \sin(x) \sin(3x) = 1$

(11) Express **without using trigonometric or inverse trigonometric functions**.

(a) (**2 Pts**)  $\cos(2\text{Cos}^{-1}(x)) =$

(b) (**2 Pts**)  $\sin(2\text{Sin}^{-1}(x)) =$

(c) (**2 Pts**)  $\tan(2\text{Tan}^{-1}(x)) =$

(12) Consider the triangle  $ABC$  with the distances  $BC = a$ ,  $AC = b$  and  $AB = c$  and the angles in  $A$ ,  $B$  and  $C$  are respectively  $\alpha$ ,  $\beta$  and  $\gamma$ .

(a) (3 Pts) State the **law of cosines** for the triangle  $ABC$ .

(b) (3 Pts) State the **law of sines** for the triangle  $ABC$ .

(c) (3 Pts) **Solve the triangle**  $ABC$  given that  $a = 30$ ,  $b = 35$  and  $\alpha = 60^\circ$

(13) (4 Pts) Write  $16x - 12y = 4y^2 + 1$  in the form  $x - h = \frac{1}{4c}(y - k)^2$ .

(14) (3 Pts) Find the **shortest distance** from the point  $(4, -1)$  to the line  $y = 2x - 3$ .

(15) Consider the function  $f(x) = 3 \cos\left(\frac{\pi}{2}x - \pi\right) - 2$ .

(a) (2 Pts) Find the **amplitude**:

(b) (2 Pts) Find the **period**:

(c) (2 Pts) Find the **phase shift**:

(d) (2 Pts) Determine **the starting point of one cycle** of the graph:

(e) (2 Pts) Determine **the ending point of one cycle** of the graph:

(f) (8 Pts) Find the **five key points** to the graph of  $y = 3 \cos\left(\frac{\pi}{2}x - \pi\right)$  and **graph one cycle** of  $y = 3 \cos\left(\frac{\pi}{2}x - \pi\right)$  and  $y = 3 \cos\left(\frac{\pi}{2}x - \pi\right) - 2$

