

**UNIVERSITY OF PUERTO RICO
RIO PIEDRAS CAMPUS
DEPARTMENT OF MATHEMATICS**

**ALGEBRA
PH.D. QUALIFYING EXAM
SYLLABUS**

The qualifying exam in Algebra is based on the material in Modern Algebra I (MATE 6201), Modern Algebra II (MATE 6202), and Linear Algebra (MATE 6150). The exam includes the following topics:

Abstract Algebra

Elementary set theory. Groups, subgroups, normal subgroups, homomorphisms, quotient groups, automorphisms, groups acting in sets, Sylow theorems and applications, finitely generated Abelian groups. Examples: permutation groups, cyclic groups, dihedral groups, matrix groups. Basic properties of rings, units, ideals, homomorphisms, quotient of rings, prime and maximal ideals, field of fractions, Euclidean domains, principal ideal domains and unique factorization domains, polynomial rings. Elementary properties of finite field extensions and roots of polynomials, finite fields. Introduction to module theory.

Linear Algebra

Matrices, linear transformations, change of basis, nullity-rank theorem, eigenvalues and eigenvectors; determinants, characteristic and minimal polynomials, Caley-Hamilton theorem; diagonalization and triangularization of operators; Jordan normal form, Rational Canonical Form, invariant subspaces and canonical forms; inner product spaces, hermitian and unitary operators, adjoints. Quadratic forms. Introduction to Tensor Product.

There will be nine problems in the exam.