UNIVERSITY OF PUERTO RICO
RIO PIEDRAS CAMPUS
DEPARTMENT OF MATHEMATICS

February 12, 2008 **Probability and Statistics II**

It will be marked the best 5 exercises.

1. Let X be a random variable with moment generating function: $M_X(t), -r < t < r$. Prove that: a)
$$Pr(X \geq a) \leq \exp(-at)M_X(t), 0 < t < r$$
and b)
$$Pr(X \leq a) \leq \exp(-at)M_X(t), -r < t < 0.$$

2. Let $[x_1, \ldots, X_n]$ a random sample from the pdf
$$f(x|\mu) = \exp[-(x-\mu)], \text{ where } -\infty < \mu < x < \infty.$$
a) Does this pdf belongs to the Exponential Family? b) Find a complete sufficient statistics.

3. Consider the hierarchical model:
$$X_i \sim \text{Normal}(\theta_i, \sigma^2), i = 1, \ldots, n, \text{independent}$$
and
$$\theta_i \sim \text{Normal}(\mu, \tau^2), i = 1, \ldots, n, \text{independent},$$
where σ^2 and τ^2 are known. a) Calculate the marginal of the x_i's, i.e integrate the θ_i's b) Are the X_i's marginally independent?

4. a)Provide the assumptions needed about $f(x|\theta)$to prove that
$$\int f(x|\theta)(\frac{\partial}{\partial\theta}\log f(x|\theta))^2 dx = -\int f(x|\theta)(\frac{\partial^2}{\partial\theta^2}\log f(x|\theta))dx.$$
b) When this holds, how can the Fisher Information may be defined?

5. Let Z and W iid, Normal Standard. Let $Y = min(Z, W)$. a) Find the density of Y^2. b) Does this have any relationship with a Chi-Square?

6. Let $[X_1, \ldots, X_n]$, a sample from

$$f(x|\theta) = 1/\theta, 0 \leq x \leq \theta, \theta > 0.$$

a) Estimate θ by Maximum Likelihood and by the method of moments. b) Calculate means and variances of both estimators. Which is better? c) assume a uniform prior for θ in the positive line. c.1) Compute the posterior density for θ c.2) For Quadratic Loss what is the optimal Bayes Estimator?