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April 25, 2002 **Data Structure II**

SOLVE EXACTLY THREE OUT OF THE FOLLOWING FIVE PROBLEMS:

1. Suppose $T_1(N) = O(f(N))$ and $T_2(N) = O(f(N))$ which of the following are true?

a. $T_1(N) + T_2(N) = O(f(N))$

b. $T_1(N) - T_2(N) = o(f(N))$

c. $\frac{T_1(N)}{T_2(N)} = O(1)$

d. $T_1(N) = \Theta(T_2(N))$

e. $T_1(N) = O(T_2(N))$

f. $T_1(N) = O(T_2(N))$

g. $T_1(N) \cdot T_2(N) = O(f(N) \cdot f(N))$

h. $(T_1(N) + T_2(N))/N = \ o(f(N))$

i. $T_1(N) = \Omega(T_2(N))$

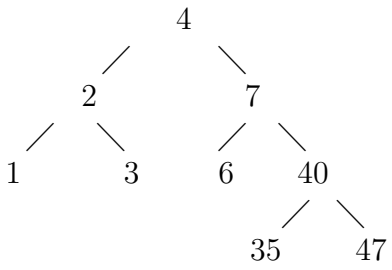
j. $f(N) = \Omega(T_1(N))$

2. For the following program segment, give an analysis of running time (in Big O notation).

```
sum = 0;
for (i = 0;  i < n;  i++)
    for (j = 0;  j < n * n;  j++)
        if ((j % i) == 0)
            sum  ++;
```

where j % i is the remainder of j divided by i.

3. Give the diagram of the AVL trees when 30 29 28 27 is inserted into the following AVL tree



4. Give a flow chart of the Depth-First Search algorithm of a graph.

5. Suppose a binary tree has leaves $l_1, l_2 \cdot \cdot l_m$ at depth $d_1, d_2 \cdots d_m$, respectively. Prove that

$$\sum_{i=1}^{M} 2^{-l_i} \leq 1$$

and determine when the equality is true.