Addition and Multiplication Properties of Real Numbers

Axioms of Equality

For all real numbers $a, b,$ and $c$:

**Reflexive Property**
$a = a$

**Symmetric Property**
If $a = b$, then $b = a$.

**Transitive Property**
If $a = b$ and $b = c$, then $a = c$.

Substitution Axiom

If $a = b$, then in any statement involving $a$ we may substitute $b$ for $a$ and obtain another true statement.

Axioms of Addition

**Closure**
For all real numbers $a$ and $b$, $a + b$ is a unique real number.

**Associative**
For all real numbers $a, b,$ and $c$, $(a + b) + c = a + (b + c)$.

**Additive Identity**
There exists a unique real number $0$ (zero) such that $a + 0 = 0 + a = a$ for every real number $a$.

**Additive Inverses**
For each real number $a$, there exists a real number $-a$ (the opposite of $a$) such that $a + (-a) = (-a) + a = 0$.

**Commutative**
For all real numbers $a$ and $b$, $a + b = b + a$.

Axioms of Multiplication

**Closure**
For all real numbers $a$ and $b$, $ab$ is a unique real number.

**Associative**
For all real numbers $a, b,$ and $c$, $(ab)c = a(bc)$.

**Multiplicative Identity**
There exists a unique nonzero real number $1$ (one) such that $1 \cdot a = a \cdot 1 = a$.

**Multiplicative Inverses**
For each nonzero real number $a$, there exists a real number $\frac{1}{a}$ (the reciprocal of $a$) such that $a \left( \frac{1}{a} \right) = \left( \frac{1}{a} \right) a = 1$.

**Commutative**
For all real numbers $a$ and $b$, $ab = ba$.

The Distributive Axiom of Multiplication over Addition

For all real numbers $a, b,$ and $c$, $a(b + c) = ab + ac$.

Definition: For all real numbers $a,b$: 
