UNIVERSITY OF PUERTO RICO
FACULTY OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS

COLLOQUIUM

The Graph Complement Conjecture: Nordhaus-Gaddum type problems for minimum rank and Colin de Verdière parameters

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Abstract: A (simple, undirected) graph describes the family of $n \times n$ real symmetric matrices $A = [a_{uv}]$ by using the edges of the graph to describe the position of nonzero off-diagonal entries of A, with the edge uv associated with the entry a_{uv} being nonzero. The minimum rank of a graph is the minimum of the ranks of the real symmetric matrices described by the graph. The problem of determining minimum rank has been investigated by many researchers over the past fifteen years. One of the many unresolved questions on this topic is the *Graph Complement Conjecture* or GCC. This conjecture asks for an upper bound on the sum of the minimum rank of a graph and the minimum rank of its complement, and may be classified as a Nordhaus-Gaddum type problem for the graph parameter minimum rank, because it involves bounding the sum of the graph parameter evaluated on a graph and on its complement. The conjectured bound is the order of the graph plus two.

Having the minimum rank of G and its complement $\overline{G}$ sum to at most $|G| + 2$ is equivalent to having the maximum nullity of G and $\overline{G}$ sum to at least $|G| - 2$ The Colin de Verdière number $\mu(G)$, which is used to characterize planarity, is the first of several graph parameters that maximize nullity among real symmetric matrices that satisfy the Strong Arnold Hypothesis (called *Colin de Verdière type parameters*); ν, which requires the matrices be positive semidefinite in addition to the Strong Arnold Hypothesis, is another. Colin de Verdière type parameters bound the maximum nullity from below, and thus bound the minimum rank from above. Variants of the GCC include the GCC for μ, $\mu(G) + \mu(\overline{G}) \geq |G| - 2$ and the GCC for ν, $\nu(G) + \nu(\overline{G}) \geq |G| - 2$. Partial results and other evidence for these various conjectures will be discussed. as well as bounds on the Nordhaus-Gaddum upper multiplier b_n for ν, where $\nu(G) + \nu(\overline{G}) \leq b_n|G|$.

Friday, February 22, 2013
Hour: 10:00 am
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