UNIVERSITY OF PUERTO RICO
RIO PIEDRAS CAMPUS
FACULTY OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS

THE DISTANCE-3 GRAPH OF THE BIGGS–SMITHS GRAPH

Italo J. Dejter

Abstract

The distance-3 graph $\mathcal{S}^3$ of the Biggs-Smith graph $\mathcal{S}$ is shown to be: **(a)** a connected edge-disjoint union of 102 tetrahedra (copies of K_4) and as such the K_4-ultrahomogeneous Menger graph of a self-dual (102_4)-configuration; **(b)** a union of 102 cuboctahedra, (copies of $L(Q_3)$), with no two such cuboctahedra having a common chordless 4-cycle; **(c)** not a line graph. Moreover, $\mathcal{S}^3$ is shown to have a $\mathcal{C}$-ultrahomogeneous property for $\mathcal{C} = \{K_4\} \cup \{L(Q_3)\}$ restricted to preserving a specific edge partition of $L(Q_3)$ into 2-paths, with each triangle (resp. each edge) shared by two copies of $L(Q_3)$ plus one of K_4 (resp. 4 copies of $L(Q_4)$). Both the distance-2 and distance-4 graphs, $\mathcal{S}^2$ and $\mathcal{S}^4$, of $\mathcal{S}$ appear in the context associated with the above mentioned edge partition. This takes us to ask whether there are any non-line-graphical connected K_4-ultrahomogeneous Menger graphs of self-dual (n_4)-configurations that are edge-disjoint unions of several copies of K_4, for positive integers $n \notin \{42, 102\}$.

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11:00-12:00