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FRACTIONAL GAUSSIAN ESTIMATES AND HOLOMORPHY OF SEMIGROUPS

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Let $\Omega \subset \mathbb{R}^N$ be an arbitrary open set and denote by $(e^{-t(-\Delta)^s_{\mathbb{R}^N}})_{t\geq 0}$ (where $0 < s < 1$) the semigroup on $L^2(\mathbb{R}^N)$ generated by the fractional Laplace operator. In the first part of the thesis we show that if T is a self-adjoint semigroup on $L^2(\Omega)$ satisfying a fractional Gaussian estimate in the sense that $|T(t)f| \leq Me^{-bt(-\Delta)^s_{\mathbb{R}^N}}|f|$, $0 \leq t \leq 1$, $f \in L^2(\Omega)$, for some constants $M \geq 1$ and $b \geq 0$, then T defines a bounded holomorphic semigroup of angle $\frac{\pi}{2}$ that interpolates on $L^p(\Omega)$, $1 \leq p < \infty$. Additionally, if T_0 is a semigroup on $C_0(\Omega)$ such that $T_0(t)f = T(t)f$ for $f \in C_0(\Omega) \cap L^2(\Omega)$ we prove that the same result also holds on the space $C_0(\Omega)$. If Ω is bounded then the same conclusion holds by $C(\overline{\Omega})$. Also, we apply the above results to the realization of fractional order operators with the exterior Dirichlet conditions.