On the Approximate Controllability of Fractional Differential Equations

Dissertation
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Abstract

We study the approximate and mean approximate controllability properties of fractional partial differential equations associated with the so-called Hilfer type time-fractional derivative and a non-negative selfadjoint operator A_B with a compact resolvent on $L^2(\Omega)$, where $\Omega \subset \mathbb{R}^N$ $(N \geq 1)$ is a bounded open set. More precisely, we show that if $0 \leq \nu \leq 1$ and $0 < \mu \leq 1$, then the system

$$\mathbb{D}_t^{\mu,\nu} u + A_B u = f\chi_\omega \ \text{ in } \ \Omega \times (0,T), \ (\mathbb{I}_t^{(1-\nu)(1-\mu)} u)(\cdot,0) = u_0 \text{ in } \ \Omega,$$

is approximately controllable for any $T > 0$, $u_0 \in L^2(\Omega)$ and any non-empty open set $\omega \subset \Omega$. In addition, if the operator A_B has the unique continuation property, then the system is also mean approximately controllable. The operator A_B can be the realization in $L^2(\Omega)$ of a symmetric, non-negative uniformly elliptic second order operator with Dirichlet or Robin boundary conditions, or the realization in $L^2(\Omega)$ of the fractional Laplace operator $(-\Delta)^s$ $(0 < s < 1)$ with the Dirichlet exterior condition, $u = 0$ in $\mathbb{R}^N \setminus \Omega$, or the nonlocal Robin exterior condition, $\mathscr{N}^s u + \beta u = 0$ in $\mathbb{R}^N \setminus \overline{\Omega}$.