Value distribution of elementary symmetric polynomials over finite fields

DISSERTATION
DEPARTMENT OF
MATHEMATICS

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June 18, 2020

Abstract

In this work we present two studies related to exponential sums and Walsh transformations of q-ary functions. In our first study we establish the asymptotic behavior of some generating functions related to the exponential sum over finite fields of elementary symmetric functions and their perturbations. This asymptotic behavior allows us to calculate the probability generating function of the probability that the elementary symmetric polynomial of degree k (and its perturbations) returns $\beta \in F_q$ where F_q represents the field of q elements. Our study extends many of the results known for perturbations over the binary field to any finite field. In particular, we establish when a particular perturbation is asymptotically balanced over a prime field and provide a construction to find such perturbations over any finite field.

Our second study is related to generalized p-ary functions. In particular, we show that Walsh-Hadamard transformations of generalized p-ary functions whose components are symmetric, rotation symmetric or a combination or concatenation of them are C-finite sequences. This result generalized many of the known results for regular p-ary functions. We also present a study of the roots of the characteristic polynomials related to these sequences and show that properties like balancedness and being bent are not shared by the underline p-ary functions.