Workshop on Dynamics, Control and Numerics for Fractional PDEs

December 5-7, 2018

University of Puerto Rico, Río Piedras
College of Natural Sciences
Department of Mathematics
Location: Embassy Suites Hotel, Isla Verde, Carolina

Organizing Committee

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KEYNOTE SPEAKERS

J.A. Burns. **High Order Approximations for Modeling and Control of PDE Systems**.

Z-Q. Chen. **Fundamental solutions to time fractional Poisson equations**.

M. Gunzburger. **Analysis and approximation of nonlocal anomalous diffusion models with application to obstacle and coefficient identification problems**.

I. Lasiecka. **Mathematical theory of evolutions arising in flow structure interactions**.

R.C. Smith. **Sensitivity Analysis, Uncertainty Quantification, and Control Design for Smart Material Systems**.

E. Zuazua. **Control of some models in population dynamics**.

High Order Approximations for Modeling and Control of PDE Systems

John A. Burns and James Cheung

Interdisciplinary Center for Applied Mathematics

Virginia Tech

Abstract: In this talk we consider higher order hp -refinement methods for approximating control systems governed by convection diffusion equations. These adaptive high order methods can be employed to construct low order approximations of the Riccati partial differential equations that arise in linear quadratic control and optimal state estimation. Under smoothness conditions we establish convergence of the approximating Riccati operators to the infinite dimensional Riccati operators that define optimal LQR and LQG controllers. This work is motivated by applications to sensor location problems for dynamic sensor systems. We employ hp - methods to both improve accuracy and to construct reduced order models suitable for real time computing. Employing high order methods one can obtain accuracy with fewer degrees of freedom than can be achieved with mesh refinement alone. Thus, this approach may be viewed as a model reduction method for which there are rigorous error bounds. Numerical examples are provided to illustrate the advantages of higher order methods for optimal control and model reduction.

*This work was supported in part by a gift from the Climate, Controls and Security Division of the United Technologies Corporation and DARPA under contract N660011824030

Fundamental solutions to time fractional Poisson equations

Zhen-Qing Chen

University of Washington

Abstract: Time-fractional diffusion equations have been actively studied in several fields including mathematics, physics, chemistry, engineering, hydrology and even finance and social sciences as they can be used to model the anomalous diffusions exhibiting subdiffusive behavior, due to particle sticking and trapping phenomena. In this talk, I will report some recent progress in the study of general fractional-time parabolic equations of mixture type, including existence and uniqueness of the solutions and their probabilistic representations in terms of the corresponding inverse subordinators with or without drifts. Sharp two-sided estimates on the fundamental solution will be given. I will then talk about fractional-time parabolic equations with source term. In particular, a new representation formula for the solution of time fractional Poisson equation will be presented, which does not involve fractional time derivative of the fundamental solution.

Analysis and approximation of nonlocal anomalous diffusion models with application to obstacle and coefficient identification problems

Max Gunzburger

Florida State University

Abstract: We consider integral equation models for anomalous diffusion, models that include as special cases fractional derivative models. We provide a brief review of a nonlocal vector calculus that facilitates (in the same manner as does the classical vector calculus for standard diffusion problems) the definition and analysis of weak formulations and finite element methods for steady-state and time-dependent problems. We illustrate the effects of nonlocality through consideration of nonlocal obstacle and coefficient identification problems.

Mathematical theory of evolutions arising in flow structure interaction

Irena Lasiecka

University of Memphis

Abstract: An appearance of flutter in oscillating structures is an endemic phenomenon. Most common causes are vibrations induced by the moving flow of a gas (air, liquid) which is interacting with the structure. Typical examples include: turbulent jets, vibrating bridges, oscillating facial palate at the onset of apnea. In the case of an aircraft it may compromise its safety. The intensity of the flutter depends heavily on the speed of the flow (subsonic, transonic or supersonic regimes). Thus, reduction or attenuation of flutter is one of the key problems in aeroelasticity with application to a variety of fields including aerospace engineering, structural engineering, medicine and life sciences.

Mathematical models describing this phenomenon involve strongly coupled systems of partial differential equations (Euler Equation and nonlinear plate equation) with interaction at the interface - which is the boundary surface of the structure. The analysis of the model leads to consideration of nonlocal PDE's. Of particular interest are models with mixed boundary conditions [such as Kutta-Joukovsky boundary conditions] whihc lead to a plethora of open problems in elliptic theory and related Hilbert-Riesz transform theory

This talk aims at providing a brief overview of recent developments in the area along with a presentation of some recent advances addressing the issues of mixed boundary conditions arising in modeling of panels uttering in a non-viscous environment. Since the properties of the flutter depend heavily on the speed of the flow (subsonic, transonic or supersonic), it is natural that the resulting mathematical theories will be very different in the subsonic and supersonic regimes. In fact, supersonic flows are known for depleting ellipticity from the corresponding static model. Thus, both wellposedness of

finite energy solutions and long time behavior of the model have been open questions in the literature. The results presented include: generation of a dynamical system associated with the model. Existence of global and finite dimensional attracting sets for the elastic structure in the absence of mechanical dissipation. Strong convergence to multiple equilibria for the subsonic model. As a consequence, one concludes that the supersonic flow alone (without any dissipation added to the elastic structure) provides some stabilizing effect on the plate by reducing asymptotically its dynamics to a finite dimensional structure. However, the resulting "dynamical system" may exhibit a chaotic behavior which however may be controlled by finite dimensional controls.

Sensitivity Analysis, Uncertainty Quantification, and Control Design for Smart Material Systems

Ralph C. Smith

North Carolina State University

Abstract: This presentation will focus on concepts pertaining to sensitivity analysis (SA), uncertainty quantification (UQ), and control design for smart materials and adaptive structures. Pertinent issues will first be illustrated in the context of applications utilizing piezoelectric and shape memory alloy actuators, finite-deformation viscoelastic models, a fractional-order model for viscoelastic materials, and quantum-informed continuum models. The use of data, to improve the predictive accuracy of models, is central to uncertainty quantification so we will next provide an overview of how Bayesian techniques can be used to construct distributions for model inputs. The discussion will subsequently focus on computational techniques to propagate these distributions through complex models to construct prediction intervals for statistical quantities of interest such as expected displacements in macro-fiber composites and strains in SMA tendons. The use of sensitivity and active subspace analysis to isolate critical model inputs and reduce model complexity is synergistic with uncertainty quantification and will be discussed next. The presentation will conclude with discussion detailing how uncertainty quantification can be used to improve robust control designs for smart material systems.

Control of some models in population dynamics

Enrique Zuazua

Deustotech-Bilbao (Spain)
Universidad Autónoma de Madrid (Spain)

Abstract: This lecture is devoted to present recent joint work in collaboration with C. Pouchol, E. Trélat and J. Zhu on the control of a bistable reaction-diffusion arising in the modelling of bilingual populations. We first analyse the possibility of controlling the system by tuning the Allee parameter to later consider in more detail the boundary control problem through a careful analysis of the phase portrait of steady states. We shall also present some recent work in collaboration with D. Maity and M. Tucsnak (Univ. Bordeaux) on a linear system in population dynamics involving age structuring and spatial diffusion (of Lotka-McKendrick type).

References

[1] E. Trélat, J. Zhu, E. Zuazua, Allee optimal control of a system in ecology, Mathematical Models and Methods in Applied Sciences, Vol. 28, No. 9 (2018) 1665 –1697.

[2] C. Pouchol, E. Trélat, E. Zuazua, Phase portrait control for 1D monostable and bistable reaction-diffusion equations, prepint, 2018 (hal-01800382).

[3] D. Maity, M. Tucsnak, E. Zuazua, Sharp Time Null Controllability of a Population Dynamics Model with Age Structuring and Diffusion, prepint, 2018 (hal-01764865).

INVITED TALKS

H. Antil. **Fractional PDEs: Control and Applications**.

U. Biccari. **Controllability of a one-dimensional fractional heat equation: theoretical and numerical aspects**.

M. D'Elia **Nonlocal Models with Nonstandard Interaction Domains: comparative analysis and efficient finite element methods**.

C.G. Gal. **On overview of well-posedness and regularity results for nonlocal in time PDEs**.

M. Parks. **Subsurface Applications for Peridynamics**.

R. Ponce. **A posteriori error estimates and maximal regularity for approximations of nonlinear fractional problems in Banach spaces**.

P. Radu. **Double nonlocality in continuum mechanics**.

L. Tebou. **Some contributions to the simultaneous and indirect stabilization of multi-component systems**.

P.R. Stinga. **How to approximate the fractional Laplacian by the fractional discrete Laplacian**.

Fractional PDEs: Control and Applications

Harbir Antil

George Mason University

Abstract: Fractional calculus and its application to anomalous transport has recently received a tremendous amount of attention. In these studies, the anomalous transport (of charge, tracers, fluid, etc.) is presumed attributable to long-range correlations of material properties within an inherently complex, and in some cases self-similar, conducting medium. Rather than considering an exquisitely discretized (and computationally explosive) representation of the medium, the complex and spatially correlated heterogeneity is represented through reformulation of the PDE governing the relevant transport physics such that its coefficients are, instead, smooth but paired with fractional-order space derivatives.

In this talk we will describe how to incorporate nonhomogeneous boundary conditions in fractional PDEs. We will cover from linear to quasilinear fractional PDEs. New notions of optimal control, inverse problem, and optimization under uncertainty will be presented. We will conclude the talk with an approach that allows the fractional exponent to be spatially dependent. This has enabled us to define novel Sobolev spaces and their trace spaces. Several applications in: imaging science, quantum random walks, geophysics, and manifold learning (data analysis) will be discussed.

Controllability of a one-dimensional fractional heat equation: theoretical and numerical aspects

Umberto Biccari

Deustotech-Bilbao (Spain)

Abstract: We analyze the controllability problem for the following one-dimensional heat equation

$$\begin{cases} z - t + (-d_x^2)^s z = g\mathbf{1}_\omega, & (x,t) \in (-1,1)\times(0,T) \\ z = 0, & (x,t) \in (-1,1)^c \times (0,T) \\ z(x,0) = z_0(x), & x \in (-1,1) \end{cases}$$

involving the fractional Laplacian $(-d_x^2)^s$, $s \in (0,1)$, on the interval $(-1,1)$. Using classical results and techniques, we show that, acting from an open subset $\omega \subset (-1,1)$, the problem is null-controllable for $s > 1/2$ and that for $s \leq 1/2$ we only have approximate controllability. This result will be confirmed by numerical experiments: employing the penalized Hilbert Uniqueness Method, joint with a finite element scheme for the approximation of the solution to the corresponding elliptic equation, we will deal with the numerical approximation of the controls. The expected controllability properties will then be deduced also at the discrete level by analyzing the behavior, with respect to the mesh size parameter, of certain quantities of interest such as the cost of controllability, the size of the final target and the optimal energy.

Nonlocal Models with Nonstandard Interaction Domains: comparative analysis and efficient finite element methods

Marta D'Elia

Sandia National Laboratories

Abstract: Fractional models and more general nonlocal models with finite-range interactions recently gained popularity in several diverse scientific and engineering applications that range from continuum mechanics to stochastic processes. We are particularly interested in nonlocal diffusion operators, i.e. the nonlocal counterpart of elliptic operators for partial differential equations (PDEs) that can be used to describe a large class of applications including nonlocal elasticity, subsurface flow, image processing and nonlocal heat conduction. These models can capture features of the solution that cannot be described by classical PDEs. However, their accuracy comes at a price: the discretization of nonlocal models usually involves dense or full matrices that require a lot of memory storage and whose assembling can be prohibitively expensive. Standard nonlocal models with finite-range interactions use euclidean (or 2-norm) balls as interaction regions. In this work we consider a novel concept of neighborhood where the interaction regions are more general so-called nonstandard interaction sets including, e.g., norm balls which are induced by the infinity- or 1-norm. Initially motivated by computational challenges, this approach can be considered an applicable model in its own right. We present an analysis of the well-posedness of the nonlocal model induced by nonstandard interaction regions and a careful comparison of models induced by different norms. Also, we illustrate our theoretical results with several two-dimensional numerical tests and present an application to image processing, where the use of infinity-balls is induced by the nature of the problem.

On overview of well-posedness and regularity results for nonlocal in time PDEs

Ciprian G. Gal

Florida International University

Abstract: We will discuss a number of recent approaches and results for nonlocal in time PDEs where the nonlocal in time derivative of the solution is expressed as a convolution with an appropriate singular kernel. The corresponding problem can be divided essentially in two different classes, that include equations of super-diffusive type (i.e., fractional in time wave equations) and equations of sub-diffusive type, respectively (i.e., fractional in time parabolic equations).

Subsurface Applications for Peridynamics

Michael Parks

Sandia National Laboratories

Abstract: Peridynamics is a nonlocal reformulation of continuum mechanics that is suitable for representing fracture and failure, see [1, 2] and the references therein. Better understanding and control of the subsurface is important to the energy industry for improving productivity from reservoirs. We motivate and explore two relevant subsurface applications for peridynamics. The first involves solving inverse problems in heterogeneous and fractured media, which may be useful in characterizing subsurface stress-state conditions [3]. The second involves the study of fracture initiation and growth from propellant-based stimulation of a wellbore [4]. Simple models and proof-of-concept numerical studies are presented.

[1] S. A. Silling, Reformulation of elasticity theory for discontinuities and long-range forces, J. Mech. Phys. Solids 48, (2000), 175-209.

[2] S. Silling, M. Epton, O. Weckner, J. Xu, and E. Askari, Peridynamic states and constitutive modeling, J. Elasticity 88, (2007), 151-184.

[3] D. Turner, B. van Bloemen Waanders and M.L. Parks, Inverse problems in heterogeneous and fractured media using peridynamics, Journal of Mechanics of Materials and Structures, 10(5), pp. 573-590, 2015.

[4] R. Panchadhara, P.A. Gordon, and M.L. Parks, Modeling propellant-based stimulation of a borehole with peridynamics, International Journal of Rock Mechanics and Mining Sciences, 93, pp. 330-343, 2017.

A posteriori error estimates and maximal regularity for approximations of nonlinear fractional problems in Banach spaces

Eduardo Cuesta and Rodrigo Ponce

Universidad de Talca (Chile)

Abstract: In this talk we consider the nonlinear fractional problem

$$u(t) = u_0 + \partial_t^{-\beta} Au(t) + F(u(t)), \quad 0 \leq t \leq T, \tag{1}$$

where $\partial_t^{-\beta} g(t)$ represents, for $g : (0, +\infty) \to X$ and $1 < \beta < 2$, the fractional integral of order $\beta > 0$ in the variable t of g and X is a Banach space. Let $\{U_n\}_{n=1}^N$ a time discretization of (1) at time levels $0 = t_0 < t_1 < t_2 < \ldots < t_N = T$, where U_n stands for the approximation to the continuous solution $u(t)$ in each t_n, i.e. $U_n \approx u(t_n)$, $1 \leq n \leq N$, and denote $I_n = [t_{n-1}, t_n]$, for $1 \leq n \leq N$. A lot of time discretizations of (1) have been accurately studied in the literature, e.g. convolution quadrature based methods, numerical inversion of the Laplace transform, collocations methods, Adomian decomposition methods, and so many others.

Let $\mathcal{U} : [0, T] \to X, \quad \mathcal{U} \in \mathcal{C}^1((0, T), X)$ be a continuous piecewise polynomial function such that, for $1 \leq n \leq N, \mathcal{U}|_{I_n} \in \mathbb{P}_3(I_n, X)$ (a polynomial of degree 3 on I_n), $\mathcal{U}(t_n) = U_n$ and $\mathcal{U}'|_{I_n}(t_n) = \mathcal{U}'|_{I_{n+1}}(t_n)$, $n \leq N - 1$, and $\mathcal{U}'(0) = \mathcal{U}'(T) = 0$.

It is a well known fact that time discretization methods for integro/differential equations produce an error between the discrete and the exact solution of these equations. Usually, the error estimate has the form $\|u - u_h\| \leq C(h)$ where u is the exact solution of the integro/differential equation, u_h is the approximated solution, h is the approximation parameter, and the function $C(h)$ (which depends, among other, of the parameter h) corresponds to the *error estimates.* Basically, there are two types of error estimations. In the first one, called *priori error estimates,* the function $C(h)$ depends on the exact solution, but not on the approximated solution, and therefore, it can be evaluated (in theory, but not possibly in practice) before computing the exact solution. On the other hand, in the second one, known as *a posteriori error estimates,* the error depends on the approximated solution but not the exact solution.

Let $e : [0, T] \to \mathcal{B}$ be the error function defined as $e(t) := \mathcal{U}(t) - u(t)$, where u is the solution to (1) and $\mathcal{U}(t_n) = U_n$ is the approximation to the continuous solution $u(t)$ in each t_n. Under some suitable conditions, in this talk, we give a comparative between the continuous and the discrete time solution and we obtain a posteriori error estimate for the nonlinear equation (1).

The second author was supported partially by the program *Beca Interamericana para jóvenes profesores e investigadores, Banco Santander.*

Double nonlocality in continuum mechanics

Petronela Radu

University of Nebraska-Lincoln

Abstract: Nonlocal models have seen a rapid resurgence in recent years, motivated by successes and promising advances in the theory of peridynamics (introduced by Stewart Silling in 2000), applications in image processing, mixing alloys, biology and many more fields. In this talk I will discuss a particular type of problems that involve double nonlocality, expressed through a double integration against different kernels. I will discuss these models that are nonlocal counterparts of second and fourth orders systems, and also their connection with classical (differential) models.

Some contributions to the simultaneous and indirect stabilization of multi-component systems

Louis Tebou

Florida International University

Abstract: Stability issues for multi-component systems involving wave, plate or beam equations with localized damping mechanisms are examined. The operator defining the damping is degenerate. First, some of the existing results in the framework of simultaneous stabilization will be reviewed; here systems having the same damping mechanism in all equations are discussed, and exponential stability as well as unique continuation results are presented. Next, the case of indirect damping mechanisms is addressed; here, the damping mechanism occurs in only one of the components of the system and the coupling should transmit it to the undamped component(s) of the system, polynomial and exponential stability results are presented.

How to approximate the fractional Laplacian by the fractional discrete Laplacian

Pablo Raúl Stinga

Iowa State University

Abstract: We use the solution to the semi-discrete heat equation in combination with the methodology of semigroups to define and obtain the pointwise formula for the fractional powers of the discrete Laplacian in a mesh of size $h > 0$. This operator corresponds to the process of a particle that is allowed to randomly jump to *any* point in the mesh with a certain probability. It is shown that solutions to the continuous fractional Poisson equation $(-\Delta)^s U = F$ can be approximated by solutions to the fractional discrete Dirichlet problem $(-\Delta_h)^s u = f$ in B_R, $u = 0$ in B_R^c. We obtain novel error estimates in the strongest possible norm, namely, the L^∞ norm, under minimal natural Hölder regularity assumptions. Key ingredients for the analysis are the regularity estimates for the fractional discrete Laplacian, which are of independent interest.

CONTRIBUTED TALKS

O. Burkovska. **Model order reduction for nonlocal problems with parametrized kernels.**

S. Charoenphon. **Vanishing relaxation time dynamics of the Jordan Moore-Gibson-Thompson (JMGT) equation arising in high frequency ultrasound (HFU).**

J. Gonzalez. **Fundamental solutions for discrete dynamical systems involving the fractional Laplacian.**

S. Guffey. **Excursions into Controllability of a Chemotaxis System via Diffusive Phenomena.**

M. Gulian. **Stochastic Solution of Parabolic and Elliptic Boundary Value Problems for the Spectral Fractional Laplacian.**

R. Khatri. **Parameter Estimation and Fractional Regularization in Tomographic Reconstruction.**

Model order reduction for nonlocal problems with parametrized kernels

Olena Burkovska and **Max Gunzburger**

Florida State University

Abstract: We consider parametrized nonlocal problems driven by spatial integral operators arising in the modeling of anomalous diffusion. In particular, we focus on kernels with nonlocal interactions limited to a ball of radius $\delta > 0$ or (truncated) fractional Laplace kernels, which are also parametrized by the fractional power $s \in (0, 1)$. We are interested in efficient and reliable evaluation of the solution for different values of the parameters. For this, we employ a parametric model order reduction approach, in particular, the reduced basis method (RBM). A major difficulty arises in the non-affinity of the integral kernel w.r.t. the parameters, which can not be directly treated by empirical interpolation due to the singularity and a lack of continuity of the kernel. This substantially affects the efficiency of the RBM. As a remedy, we propose suitable approximations of the kernel based on the parametric-regularity of the bilinear form and the improved spatial regularity of the solution [1]. Finally, we certify the RBM by providing reliable a posteriori error estimators and support the theoretical findings by numerical experiments.

References

[1] O. Burkovska and M. Gunzburger. Regularity and approximation analyses of nonlocal variational equality and inequality problems. ArXiv e-prints 1804.10282, 2018.

Vanishing relaxation time dynamics of the Jordan Moore-Gibson-Thompson (JMGT) equation arising in high frequency ultrasound (HFU)

Sutthirut Charoenphon

University of Memphis

Abstract: The (third-order in time) JMGT equation is a nonlinear (quasilinear) Partial Differential Equation (PDE) model introduced to describe the acoustic velocity potential in ultrasound wave propagation. One begins with the parabolic Westervalt equation governing the dynamics of the pressure in nonlinear acoustic waves. In its derivation from constitutive laws, one then replaces the Fourier law with the Maxwell-Cattaneo law, to avoid *the paradox of the infinite speed of propagation*. This process then gives raise to a new third time derivative term, with a small constant coefficient τ, referred to as relaxation time. As a consequence, the mathematical structure of the underlying model changes drastically from the parabolic character of the Westervelt model (whose linear part generates a s.c, analytic semigroup) to the hyperbolic-like character of the JMGT model (whose linear part generates a s.c, group on a suitable function space). It is therefore of both mathematical and physical interest to analyze the asymptotic behavior of hyperbolic solutions of the JMGT model as the relaxation parameter $\tau \geq 0$ tends to zero. In particular, it will be shown that for suitably calibrated initial data one obtains at the limit exponentially time-decaying solutions. The rate of convergence allows one then to estimate the relaxation time needed for the signal to reach the target. The interest in studying this type of problems is motivated by a large array of applications arising in engineering and medical sciences. These include applications to welding, lithotripsy, ultrasound technology, noninvasive treatment of kidney stones.

Fundamental solutions for discrete dynamical systems involving the fractional Laplacian

Jorge González-Camus

Universidad de Santiago, Chile

Abstract: In this talk we prove representation results for solutions of a time-fractional differential equation involving the discrete fractional Laplace operator in terms of generalized Wright functions. Such equations arise in the modeling of many physical systems, for example chain processes in chemistry and radioactivity. Our focus is in the problem:

$$\mathbb{D}_t^\beta u(n,t) = -(-\Delta_d)^\alpha u(n,t) + g(n,t),$$

where $0 < \beta \leq 2$, $0 < \alpha \leq 1$, $n \in \mathbb{Z}$, $(-\Delta_d)^\alpha$ is the discrete fractional Laplacian and $\mathbb{D}_t^\beta$ is the Caputo fractional derivative of order β. Also we shall present important special cases such as the discrete heat and wave equations as a direct consequence of the above mentioned representation.

Excursions into controllability of a Chemotaxis system via Diffusive Phenomena

Stephen Guffey

University of Memphis

Abstract: In this talk we introduce a coupled PDE-system describing a bacterial infection in a chronic wound. It couples 4 PDE equations, 3 of them of diffusive type, involving 4 variables. One particularly interesting mathematical feature of the model is a chemotactic reaction between two of the unknowns. Said reaction is modeled after the Keller-Segel model for chemotaxis. For such parabolic-like system, the relevant control theoretic concept is null-controllability: the ability to steer an arbitrary initial condition in the natural state space to rest, in an arbitrarily short, universal time interval, by means of a control function, in our case localized in an arbitrarily small sub-domain of the original PDE-domain. Such concept turns out to be equivalent to a more convenient observability inequality of the dual (adjoint) PDE-problem. Establishing the validity of such observability inequality is the crux of the matter. It is a challenging mathematical task. Our approach is based on the use of so-called Carleman estimates for parabolic problems, which need to be adapted to the particular coupled PDE-system at hand. For clarity, we shall illustrate how to establish observability inequalities in the simplified case of a coupled system of diffusion equations.

Stochastic Solution of Parabolic and Elliptic Boundary Value Problems for the Spectral Fractional Laplacian

Mamikon Gulian

Brown University

Abstract: We prove stochastic solution formulas for the recently established inhomogeneous Spectral fractional Laplacian with nonzero Dirichlet boundary conditions. These formulas involve subordinate stopped Brownian motion, and are established in the parabolic case using the theory of Feller semigroups and the results of Balakrishnan on fractional powers of operators. Then, we study the well-posedness, regularity, and convergence to the steady state for the fractional parabolic problem to obtain a stochastic solution formula for the corresponding fractional elliptic problem. Finally, we discuss numerical implementation and walk-on-spheres algorithms.

Role of Fractional Laplacian in Inverse Problems

Harbir Antil[*], Zichao (Wendy) Di[**], Ratna Khatri[*]

[*] George Mason University, [**] Argonne National Laboratory

Abstract: Tomographic reconstruction is a non-invasive 2D/3D image recovery technique based on inversion of a sequence of 1D/2D projections arising from multiple angles. One way of solving this problem is via linear least squares optimization formulation assuming the experimental data follows a Gaussian distribution. However, the limited data, due to the expensive and imperfect nature of the physical experiment, makes the reconstruction problem ill-posed. In this work, we propose a novel regularization for this problem: the fractional Laplacian, to improve the reconstruction quality. However, choosing an optimal regularization parameter is known to be challenging itself. We propose a neural network to efficiently learn the optimal regularization parameter. We then provide a performance comparison with the commonly used Total Variation regularization.

List of Participants

1. Harbir Antil (George Mason University)
2. Rafael Aparicio (University of Puerto Rico, Río Piedras)
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5. Umberto Biccari (Deustotech-Bilbao, Spain)
6. Marcelo Bongarti (University of Memphis)
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14. Zhen-Qing Chen (University of Washington)
15. Marta D'Elia (Sandia National Labotarories)
16. Victor Diaz-Martinez (University of Puerto Rico, Mayagüez)

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20. Jorge Gonzalez (Universidad de Santiago, Chile)

21. Stephen Guffey (University of Memphis)

22. Mamikon Gulian (Brown University)

23. Max Gunzburger (Florida State University)

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