Departamento de Matemáticas

Facultad de Ciencias Naturales
Recinto de Río Piedras

MATE 3152

Apellidos: ____________________ ____________________ Nombre: ____________________
No. de estudiante: ____________________ Profesor: ____________________
Exam II (Practice) 2 de mayo de 2011 # de sección: ____________________

Para obtener crédito muestre todo su trabajo. Explique claramente su contestación.

(1) Find the following limits if they exist in $[-\infty, \infty]$

(a) $\displaystyle\lim_{x\to 0^+} \frac{\tan x}{\sqrt{x}}$

(b) $\displaystyle\lim_{x\to 0^+} \frac{\tan x - \sin x}{x^2 \sin x}$

(c) $\displaystyle\lim_{x\to 0^+} \frac{\tan^{-1} x - x + \frac{x^3}{3}}{\pi x^5}$

(2) Determine A, B such that $\displaystyle\lim_{x\to 1} \frac{Ax^4 + Bx^3 + 1}{(x-1)\sin(\pi x)}$ exists in $\mathbb{R}$.

(3) Find the following limits if they exist in $[-\infty, \infty]$

(a) $\displaystyle\lim_{x\to 1} \left[\frac{1}{x-1} - \frac{x}{\ln x}\right]$

(b) $\displaystyle\lim_{x\to 0} \left[\csc^2 x - \frac{1}{x^2}\right]$

(c) $\lim_{x\to\infty}\left[1-\frac{\pi}{x}\right]^x$

(4) Let $p \geq 0$. Find $\lim_{n\to\infty}\frac{1^p+2^p+...+n^p}{n^{p+1}}$.

(5) Determine whether the following improper integrals are convergent or divergent. If possible, compute the integrals when they converge.

(a) $\int_{\pi}^{\infty}(e^x+e^{-2x})dx$

(b) $\int_{-\infty}^{\infty}\frac{x}{e^{|x|}}dx$

(c) $\int_{-\infty}^{\infty}\frac{x}{e^{-|x|}}dx$

(d) $\int_{-\infty}^{\infty}\frac{1}{x^2+2x+10}dx$

(e) $\int_{0}^{\infty}e^{-x}\sin x dx$

(f) $\int_{2}^{\infty}\frac{\ln x}{x^2}dx$

(g) $\int_{2}^{4}\frac{1}{\sqrt{x^2-4}}dx$

(h) $\int_{2}^{1000}\frac{1}{x(\ln x)^{120}}dx$

(i) $\displaystyle\int_0^\infty \frac{\sin x}{x} dx$ (Use integration by parts; do not compute)

(j) $\displaystyle\int_0^\pi \frac{\cos x}{\sqrt{x}(x+1)} dx$ (do not compute)

(k) $\displaystyle\int_0^\infty \frac{\sin x}{x^2} dx$

(l) $\displaystyle\int_\pi^\infty \cos(x^2) dx$ (Use integration by parts; do not compute))

(m) $\displaystyle\int_0^1 \frac{x}{\sqrt[3]{1-x^2}} dx$

(n) $\displaystyle\int_{-9}^9 \frac{x}{81-x^2} dx$

(6) (a) Find the limits if they exist.

(i) $\displaystyle\lim_{n\to\infty} (1 + \frac{\cos x}{n})^n$

(ii) $\displaystyle\lim_{n\to\infty} \frac{e^{4n}}{n^{1000} + 12n + 5}$

(iii) $\displaystyle\lim_{n\to\infty} \frac{(-\pi)^n}{7^n}$

(iv) $\displaystyle\lim_{n\to\infty} \frac{7^n}{\pi^n}$

(v) $\displaystyle\lim_{n\to\infty} \sum_{k=1}^{n} (\sin \frac{k}{n}) \frac{1}{n}$

(vi) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{\pi}{n}(\frac{3}{1+(\frac{k}{n})^2})$

(vii) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{2^k}{(2^{k+1}-1)(2^k-1)}$

(b) Say with justification whether the series below converge or diverge

(i) $\sum_{n=1}^{\infty} (\frac{e}{\pi})^n$

(ii) $\sum_{n=1}^{\infty} \frac{e^{-\sqrt{n}}}{n^{4\pi}}$

(iii) $\sum_{k=3}^{\infty} \ln(1-\frac{1}{k^2})$

(iv) $\sum_{k=3}^{\infty} \ln(1-\frac{1}{k})$

(c) Say whether the following series are convergent or divergent.

(i) $\sum_{k=1}^{\infty} \frac{k^3 \sin 1}{2k^5+k-1}$

(ii) $\sum_{k=1}^{\infty} \frac{k}{2k^2+k+11}$

(iii) $\sum_{k=1}^{\infty} k^3 \sin\frac{\pi}{k^3}$

(iv) $\sum_{k=1}^{\infty} \frac{k}{e^k}$

(v) $\sum_{k=1}^{\infty} \frac{k^3 + \sin 1}{2k^5 \ln k}$

(vi) $\sum_{k=1}^{\infty} \frac{k^3}{2 + ke^k}$

(vii) $\sum_{k=1}^{\infty} \frac{1}{k^p} \sum_{j=1}^{k} \frac{1}{j^p}$
(discuss for $p \in \mathbb{R}$)

(viii) $\sum_{n=1}^{\infty} \sqrt{n}(1 - \cos \frac{1}{n})$

(d) Discuss convergence of the following

(i) $\sum_{k=1}^{\infty} \frac{\cos k\pi}{2k - 1}$

(ii) $\sum_{k=1}^{\infty} \frac{(-1)^k k}{2k^2 + k - 1}$

(iii) $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k+1} + 3\sqrt{k}}$

(e) Radius of convergence of the power series

(i) $\sum_{k=1}^{\infty} \frac{(-1)^k k}{2k^2 + k - 1}(x - 5)^k$

(ii) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k k}{2k^2+k-1}(x-5)^k$

(iii) $\displaystyle\sum_{k=1}^{\infty} kx^k$

(iv) $\displaystyle\sum_{k=1}^{\infty} \frac{\pi^5}{k!}(x+9)^k$

(f) For the given functions, do the following

(i) Find the Taylor series about $x = c$.

(ii) Determine the radius of convergence.

(iii) Say whether the series converges to $f(x)$.

(g) $f(x) = \sin(3x), \quad c = 0$

(h) $f(x) = \sin(3x), \quad c = \pi$

(i) $f(x) = e^{3x}, \quad c = 0$

(j) $f(x) = \ln(1+3x), \quad c = 0$

(k) $f(x) = \sin^{-1}(3x), \quad c = 0$

(l) $f(x) = \dfrac{3}{2-9x}, \quad c = 2$

(m) $f(x) = \tanh^{-1} x, \quad c = 0$

(n) $f(x) = (1 + \frac{x}{3})^{-5}, \quad c = 0$

(o) $f(x) = (1 + \frac{x}{3})^{-5}, \quad c = -3$

(p) $f(x) = (1 + \frac{x}{3})^{-5}, \quad c = 4$

(7) Find a power series solution for the differential equations:

(a) $y'' + Ay = 4x + 5$ with $y(0) = B$ where A, B are given numbers.

(b) $y'' + (x-1)y = 4x + 5$ with $y(0) = 1$.

(8) Use Taylor polynomials of order 4 to approximate $f(a)$

(a) $f(x) = \sqrt[5]{32 + x}$ with $a = 33$.

(b) $f(x) = \cos x$ with $a = \frac{\pi}{12}$.