Departamento de Matemáticas

Facultad de Ciencias Naturales
Recinto de Río Piedras

MATE 3152

Apellidos: ________________ ________________ Nombre: ________________
No. de estudiante: ____________________ Profesor: ____________________
Examen #3 `16 de Mayo de 2007` # de sección: ______________

Para obtener crédito muestre todo su trabajo. Explique claramente su contestación.

Note. All answers should be justified.

(1) (24 pts) Determine if the following sequences are convergent and find the limits:

(a) $a_n = \dfrac{5n + (-1)^n}{4n}$

(b) $a_n = \dfrac{(-1)^n}{\sqrt{n}}$

(c) $a_n = n\left(\sqrt{4 - \dfrac{1}{n}} - 2\right)$

(d) $a_n = \dfrac{\ln(n+2)}{\sqrt{n+2}}$

(e) $a_n = \dfrac{2^n}{n!}$

(f) $a_n = \dfrac{e^n}{4^n + 3}$

(2) (24 pts) For each of the following series, examine whether it converges or not. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{(1+n^3)^2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \sin^2 \frac{1}{\sqrt{n}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \cos \frac{1}{n^2}$

(d) $\displaystyle\sum_{n=1}^{\infty} \sin \frac{1}{n^2}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{2+n}}$

(f) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^{3/2}}{n^{5/2}+2n-1}$

(3) (27 pts) Which of the following series are convergent. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\ln n)^{10}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{e^n+1}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{n^n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}(\ln n)^{10}}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}(\ln n)^{10}}$

(f) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n+2}{n^2+n}$

(4) (6 pts) Show that the series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n-1}{2n}\right)^n$ is convergent.

(5) (a) (3 pts) Let f be a function of class C^∞. Suppose $n \in \mathbb{N}$. Write Taylor's formula about 0 to the order n.

(b) (6 pts) Find a formula for $f^{(n)}(x)$ in each of the following cases.

(i) $f(x) = \cos x$.

(ii) $f(x) = \sin(\pi x)$

(iii) $f(x) = \dfrac{1}{2-x}$

(c) (4 pts) For each of the functions below, show that the remainder $R_n(x)$ converges to zero for x in an interval to be specified.

(i) $f(x) = \sin(\pi x)$

(ii) $f(x) = \dfrac{1}{2-x}$

(d) (4 pts) For each of the functions below, write the Taylor-MacLaurin series and give its radius of convergence.

(i) $f(x) = \sin(\pi x)$

(ii) $f(x) = \dfrac{1}{2-x}$

(6) (12 pts) Assume that the square $ABCD$ below has sides of length 1, and that E, F, G, H are the midpoints of the sides. If the indicated pattern is continued indefinitely, what will be the area of the painted region?