

Departamento de Matemáticas

Facultad de Ciencias Naturales

Recinto de Río Piedras

**MATE
3152**

Apellidos: _____ Nombre: _____

No. de estudiante: _____ Profesor: V. Keyantuo

Examen I 29 de febrero de 2012 # de sección: 002

Para obtener crédito muestre todo su trabajo. Explique claramente su contestación.

Part I [Differentiation and Inverse Functions] (24 pts)

- (1) (6 pts) Compute $\frac{d}{dx}[\sin(\frac{1}{x})]$

$\frac{d}{dx}[\sin(\frac{1}{x})]=$

- (2) (6 pts) Suppose $\tanh(y) = x$ for $|x| < 1$ and recall that $\tanh(\theta) = \frac{\sinh(\theta)}{\cosh(\theta)}$. Compute $\frac{dy}{dx}$

$\frac{dy}{dx} =$

- (3) (6 pts) Compute $\frac{d}{dx}[\tanh^{-1}(x)]$

$$\frac{d}{dx}[\tanh^{-1}(x)] =$$

- (4) (6 pts) $\frac{d}{dx} \left[\sin^{-1}\left(\frac{1}{x}\right) \right]$

$$\frac{d}{dx} \left[\sin^{-1}\left(\frac{1}{x}\right) \right] =$$

Part II [Differential Equations] (18 pts)

- (1) (8 pts) Solve the differential equation $y' = y^2 e^x + e^x$

$$y =$$

- (2) (10 pts) Solve the differential equation $y' - e^{x-y} = e^x + e^{-y} + 1$

$y =$

Part III [Partial Fractions] (24 pts)

- (1) (6 pts) Divide the polynomial $P(x) = x^3 + 3x - 5$ by $Q(x) = x^3 + 3x^2 + 2x$ and write the result as $P(x) = C(x)Q(x) + R(x)$.

$C(x) =$; $R(x) =$

- (2) (8 pts) Obtain the partial fraction decomposition for $g(x) = \frac{x^3 + 3x - 5}{x^3 + 3x^2 + 2x}$

$$g(x) = \frac{4x^3 - 1}{x^3 + 3x^2 + 2x} =$$

- (3) (10 pts) Obtain the partial fraction decomposition for $\frac{12x^2 + 18}{x^3 - 64}$

$$\frac{12x^2 + 18}{x^3 - 64} =$$

Part IV [Indefinite Integrals-1] (24 pts)

Compute the indefinite integrals:

(1) (6 pts) $\int \tan^{-1}(u) du$

$$\int \tan^{-1}(u) du =$$

(2) (6 pts) $\int \cos^3(x) dx$

$$\int \cos^3(x) dx =$$

(3) (6 pts) Compute the integral $\int e^{\sin^2(x)} \sin(2x) dx$

$$\int e^{\sin^2(x)} \sin(2x) dx =$$

- (4) (6 pts) Compute the integral $\int e^{2x} \sin(x) dx$

$$\int e^{2x} \sin(x) dx =$$

Part V [Indefinite integrals-2] (20 pts)

- (1) (10pts) Compute the integral $\int \frac{60x}{x^2 + 2x + 5} dx$

$$\int \frac{60x}{x^2 + 2x + 5} dx =$$

- (2) (10 pts) Compute the integral $\int x \ln(x) dx$

$$\int x \ln(x) dx =$$