Departamento de Matemáticas

Facultad de Ciencias Naturales
Recinto de Río Piedras

MATE 3152

Apellidos: _____ _____ Nombre: _____
No. de estudiante: _____ Profesor: V. Keyantuo
Examen IIa 30 de noviembre de 2011 # de sección: 003

Para obtener crédito muestre todo su trabajo. Explique claramente su contestación. **Note. All your work should be included in the space provided.**

Part I [Partial Fractions] (8 pts)

(1) (8 pts) Obtain the partial fraction decomposition for $g(x) = \dfrac{x^2 - 5}{x^2 + 5x - 6}$

$g(x) = \dfrac{x^2 - 5}{x^2 + 5x - 6} =$

Part II [Indeterminate Forms] (18 pts)

(1) (8 pts) $\lim_{x\to\infty}(1-\frac{2\pi}{3x})^{-\pi x}$

$\lim_{x\to\infty}(1-\frac{2\pi}{3x})^{-\pi x}=$

(2) (10 pts) Find A and B such that $\lim_{x\to 2}\frac{Ax^2+4x+B}{(x-2)\sin(\frac{\pi}{2}x)}$ exists in $\mathbb{R}$.

$A=$; $B=$; $\lim_{x\to 2}\frac{Ax^2+4x+B}{(x-2)\sin(\frac{\pi}{2}x)}=$

Part III [Indefinite Integrals] (20 pts)

(1) Compute the indefinite integral:

(a) (10 pts) $\int \frac{x^2}{4+x^6} dx$

$$\int \frac{x^2}{4+x^6} dx =$$

(b) (10 pts) $\int \sin^3(2x)\cos^2 x dx$

$$\int \sin^3(2x)\cos^2 x dx =$$

Part IV [Improper Integrals and series] (32 pts)

Determine whether the following series converges or not. *No credit for correct box checked if proper justification is not provided.*

(1) (8 pts) $\sum_{n=1}^{\infty} \frac{4}{\sqrt{n}(n+1)}$

The series: converges ☐ diverges ☐

(2) (8 pts) $\sum_{n=1}^{\infty} \ln(1+\frac{\pi}{n})$

The series: converges ☐ diverges ☐

(3) (8 pts) $\sum_{n=0}^{\infty} \frac{(-1)^n(n+4)}{n^2+n+2}$

The series: converges ☐ diverges ☐

(4) (8 pts) $\displaystyle\sum_{n=1}^{\infty} \frac{\pi^{5n}(n+2)}{n!\cos(\frac{\pi}{n})}$

The series: converges ☐ diverges ☐

Part V [Power and Taylor series] (32 pts)

(1) (7 pts) Find the radius of convergence for the power series:

$$\sum_{n=0}^{\infty} \frac{2n}{4n+1}(x-3)^n$$

$R =$

(2) (7 pts) Obtain the power series expansion for $f(x) = e^{-\pi x^2}$ about $x = 0$.

$f(x) =$

(3) (8 pts) Obtain the power series expansion for $g(x) = \dfrac{1}{\sqrt{2+x}}$ about $x = 0$.

$g(x) =$

(4) (10 pts) Let $h(x) = 243(1+2x)^{-5/2}$. Find $h^{(12)}(4)$ by using the Taylor series expansion about $x = 4$.

$h^{(12)}(4)=$