Universidad de Puerto Rico
Recinto de Río Piedras
Facultad de Ciencias Naturales
Departamento de Matemáticas
MATE 3151 – Examen final julio de 2014

Apellidos: _____ _____ Nombre _____
No. Estudiante: _____ Profesor: Sección

Instructions:

Include details of all the answers.
Use of electronic calculators or cellular phones is prohibited

(1) (10 pts.) Consider the function: $f(x) = 4x - \dfrac{12}{x}$

(a) (3 pts.) For $ah \neq 0$, simplify the expression $\dfrac{f(a+h) - f(a)}{h}$.

(b) (3 pts.) From the previous question, determine $f'(a)$ **using the definition** of the derivative.

(c) (4 pts) Provide an equation of the tangent line at $x = 2$.

(2) (6 pts). Use the definition of the derivative to compute the limit:

$$\lim_{x \to \ln 2} \frac{\ln(1 + e^{5x}) - \ln(33)}{x - \ln 2}$$

(3) (10 pts) A particle is moving on the circle $(x-1)^2 + y^2 = 25$. When it passes through the point $(5, -3)$ the $x-$coordinate is increasing at the rate of $5cm/sec$. What is the rate of change of the $y-$coordinate then?

(4) (15 pts). A particle is moving on a horizontal straight line oriented positively from left to right. The velocity of the particle is given by $v(t) = -32t + 96$. At the time $t = 0$, the particle is located 20 units to the right of the origin.

(a) (2 pts) What is the acceleration of the particle?

(b) (2 pts) Find the time-intervals when the particle is moving to the right.

(c) (2 pts) Find the time-intervals when the particle is moving to the left.

(d) (5 pts) Find an expression for the position of the particle.

(e) (2 pts) Find the displacement of the particle between $t = 0$ and $t = 6$.

(f) (2 pts) Find the distance covered by the particle between $t = 0$ and $t = 6$.

(5) (15 pts). Evaluate the following **indefinite integrals**.

(a) $\int 8\sin(4x)\,dx =$

(b) $\int 12x^2\cos^2(2x^3)\,dx =$

(c) $\int \dfrac{4x-6}{x^2-3x+1}\,dx =$

(6) (18 pts.) Compute the definite integrals

(a) $\displaystyle\int_0^5 \frac{32x^3}{1+2x^4}dx$

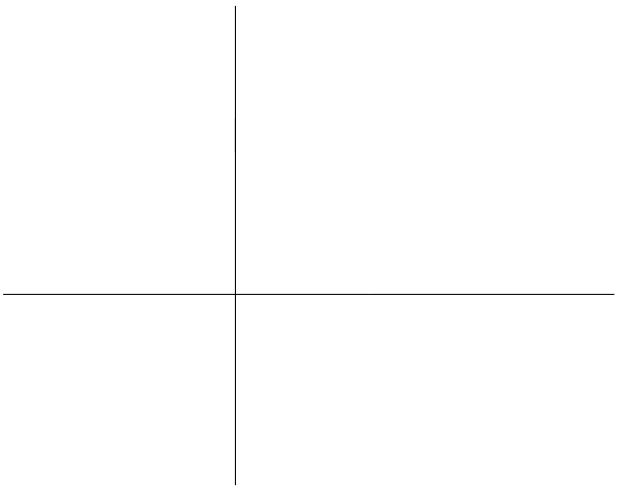
(b) $\displaystyle\int_0^{\sqrt{\pi}} x\sin(x^2)\cos(x^2)dx$

(c) $\displaystyle\int_{\ln 5}^{\ln(29)} \frac{2e^x}{\sqrt{e^x+20}}dx$

(7) (10 pts.) The *half-life* of the plutonium isotope Pu-239 is $T = 24,100$ years. Suppose that 30 grams of the plutonium isotope Pu-239 was released in the Chernobyl nuclear accident. Determine the time Z it will take for the 30 grams to decay to 5 grams. *Express Z as a function of T.*

(8) (16 pts.) Consider the region Ω bounded by the curves $y = 0$, $y = 4x$ and $y = -x + 12$.

(a) (2 pts.) Provide a sketch for the region Ω

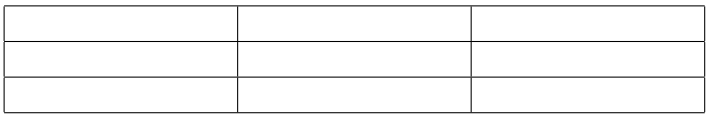


(b) (2 pts.) Compute the area of the region Ω using the familiar formula for the area of a triangle.

(c) (4 pts.) Compute the area of the region Ω using Calculus (expressing the area as a definite integral)

(d) (8 pts.) Compute the volume of the solid obtained by rotating Ω about the $x-$axis.

(9) (10 pts.) A farmer in the Midwest would like to protect a rectangular garden with a fence. At the same time, he would like to separate it into *nine* equal parts as shown in the figure below. He has L=9600m of fencing for the project. What should the **dimensions** of the garden be if he wants to enclose the *largest possible* area?



Note. $\frac{d}{dx}e^x = e^x$ and $\frac{d}{dx}\ln|x| = \frac{1}{x}$; $\frac{d}{dx}\int_a^x f(s)ds = f(x)$; $\int_a^b f(x)dx = G(b) - G(a)$ if G′(x) = f(x).

Chain Rule: $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$.

Derivative: $f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$.

$1 - 2\sin^2 x = \cos(2x)$, $-1 + 2\cos^2 x = \cos(2x)$, $\sin(2x) = 2\sin x \cos x$.

Practice

(1) (8 pts) Use derivatives to evaluate the following limits. In each case, **specify the function being used and its derivative, and the point a at which it is taken**.

(a) $\lim_{x\to\frac{1}{4}} \dfrac{\tan^3(\pi x) - 1}{x - \frac{1}{4}} =$

(b) $\lim_{x\to 2} \dfrac{(x^2+1)^5 - 3125}{x - 2} =$

(2) (8 pts). Compute the following derivatives.

(a) $\dfrac{d}{dx}\left[\left(\dfrac{e^x+3}{3-e^x}\right)^3\right] =$

(b) $\dfrac{d}{dx}\left[\sin(3-x)\ln|3x^4+9|\right] =$

(3) (10 pts) Let $f(x) = (x-2)^4 - 2(x-2)^2$.

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(a) (2 pts) Compute and factor the derivative of f

(b) (2 pts) Determine all the critical points of f

(c) (2 pts) Determine the intervals where f is increasing.

(d) (1 pts) Determine the local extreme values for f

(e) (2 pts) Determine the concavity of the graph of f

(f) (1 pts) Determine the inflection points of f (if any)