Universidad de Puerto Rico
Departamento de Matemáticas
MATE 3151– Exam III– Verano 2014– July 03, 2014

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Note. Include details of all the answers.No credit given if answer is not justified.
No cellphones or electronic calculators allowed.

(1) (**16 pts**). Let $f(x) = 2 + 12x$ for $x \in [0, 4]$. Let $P := \{0, \frac{1}{2}, 1, 2, 3, 4\}$ be a partition of $[0, 4]$, that is, $x_0 = 0,\ x_1 = \frac{1}{2},\ x_2 = 1,\ x_3 = 2,\ x_4 = 3,\ x_5 = 4$.

Use the following table to answer the questions below.

I_k	Δx_k	M_k	$M_k\Delta x_k$	m_k	$m_k\Delta x_k$	s_k	$f(s_k)$	$f(s_k)\Delta x_k$
Total		XXX		XXX		XXX	XXX	

(a) (4 pts) Evaluate the Riemann sum, using for $0 \le k \le 4$, s_k, be the **midpoint** of the interval $[x_k, x_{k+1}]$.

$\mathcal{R}(f, P) =$

(b) (4 pts) Evaluate the upper sum $U(f, P)$

$U(f, P) =$

(c) (4 pts) Evaluate the lower sum $L(f, P)$

$L(f, P) =$

(d) (4 pts) Compute the definite integral $\int_0^4 f(x)dx$

(2) (**20 pts.**) Compute the definite integrals

(a) (5 pts.) $\int_0^2 20x^4\sqrt{1+2x^5}dx$

(b) (5 pts.) $\int_0^1 (5x^{3/2} - 3x^{5/2})dx$

(c) (5 pts.) $\int_0^\pi 24\cos^2(x)\sin(x)dx$

(d) (5 pts.) $\int_0^2 \frac{\ln(x+9)}{x+9}dx$

(3) (**16 pts**). Evaluate the following **indefinite integrals**.

(a) $\displaystyle\int \frac{e^{-x}}{4+e^{-x}}\,dx =$

(b) $\displaystyle\int (4x^3+x^2)\cdot\sqrt{x}\,dx =$

(c) $\displaystyle\int 20x\sin(5x^2-1)\,dx =$

(d) $\displaystyle\int \frac{4x+6}{\sqrt{x^2+3x+1}}\,dx =$

(4) (**4 pts**) State the **First part of the Fundamental Theorem of Calculus** for a continuous function on $[a, b]$.

(5) (**12 pts**). Evaluate the following **derivatives**.

(a) $\dfrac{d}{dx}\left(\displaystyle\int_1^x \frac{9t^4 \cos t}{18 + t^3}\, dt\right) =$

(b) $\dfrac{d}{dx}\left(\displaystyle\int_\pi^4 \sin^3(t^4 + 5)\, dt\right) =$

(c) $\dfrac{d}{dx}\left(\displaystyle\int_3^{8x} \cos^2(2t)\, dt\right) =$

(6) (**8 pts**). Suppose that $\int_2^x e^{-t} f(t)dt = (2-5x^2)^5$. Use the fundamental theorem of Calculus to find

$f(x) =$

(7) (**4 pts**) State the **second part of the Fundamental Theorem of Calculus** for a continuous function on $[a, b]$.

(8) (**12 pts**). Evaluate the following **definite integrals**.

(a) $\int_0^4 \left(24\sqrt{x} + 2x + 12x^2 + 1\right)\, dx =$

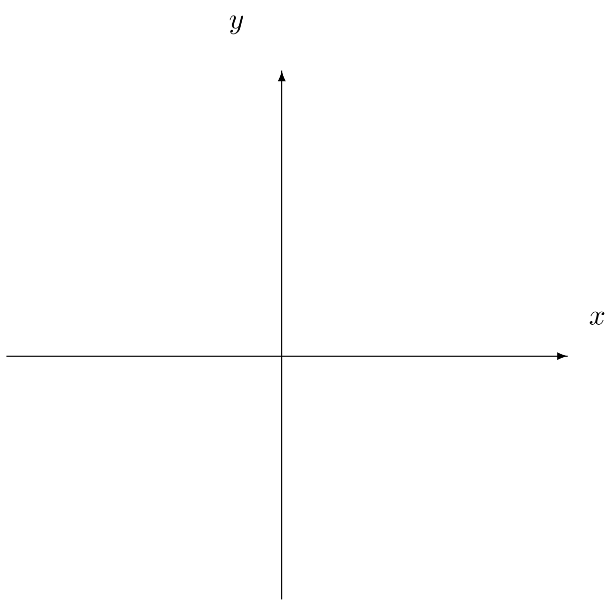
(b) $\int_0^3 \left[\frac{d}{dx}\left(\sqrt{7+x^2}\right)\right]\, dx =$

(9) (**8 pts**). Consider the function $f(x) = 9 - 12x^2$ defined on the interval $[-2, 2]$.

(a) (6 pts). Determine the **average value** of $f(x)$ on the interval $[-2, 2]$.

(b) (2 pts). Find all numbers c in the interval $(-2, 2)$ that satisfy the conclusion of the **Mean Value Theorem for Integrals**.

(10) (**10 pts**). Let Ω be the region bounded by the graphs of the functions $y = 4 - x^2$ and $y = -2x - 4$. **Sketch** the region Ω and calculate the **area** A of Ω.



$A=$

Nota. $\frac{d}{dx}e^x = e^x$ and $\frac{d}{dx}\ln|x| = \frac{1}{x}$; $\frac{d}{dx}\int_a^x f(s)ds = f(x)$; $\int_a^b f(x)dx = G(b) - G(a)$ if $G'(x) = f(x)$.

Chain Rule: $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$.