Universidad de Puerto Rico
Recinto de Río Piedras-Facultad de Ciencas Naturales
Departamento de Matemáticas
MATE 3151 – Exam II-Verano 2014- June 27, 2014

Apellidos: ______________________ ______________________ Nombre ______________________
No. Estudiante: ____ Profesor: Sección 002

Important Note: **Provide detailed answers to the questions.**

(1) (**20 pts**). Evaluate the following derivatives.

(a) $\dfrac{d}{dx}\left[\left(\dfrac{x+3}{3-x}\right)^8\right] =$

(b) $\dfrac{d}{dx}\left[\dfrac{e^{x^2}}{\cos x}\right] =$

(c) $\dfrac{d}{dx}\left[(x^3+3)\ln|x^2+1|\right] =$

(d) $\dfrac{d}{dx}\left[\tan^4\left(\sin^2(\pi x - 3)\right)\right] =$

(2) (**16 pts**) An object moves along the y-axis (**vertical axis**). Its position at each t is given by $S(t) = -16t^2 + 192t + 48$.

(a) (2 pts) Determine the formula of the **velocity** $\nu(t)$.

(b) (2 pts) Determine the time(s) at which the velocity of the object is 0.

(c) (2 pts) Determine the formula of the **acceleration** $a(t)$.

(d) (4 pts) Determine the highest altitude attained by the object.

(e) (4 pts) Determine the time τ such that $\nu(\tau) = \dfrac{S(8) - S(2)}{6}$.

(f) (2 pts) Name the theorem which justifies a priori the existence of the time τ of the previous question.

(3) (**10 pts**). Find an **equation for the tangent line** at the point $(2, 2)$ to the graph of the function y determined by the relation $x^3 - 2xy + 6y^2 = 24$.

(4) (**6 pts**). Suppose that f and g are differentiable functions such that $f(2) = -8, f'(2) = -6,\ f'(8) = -10$ and $g(2) = 8,\ g'(2) = 10,\ g'(-8) = 9$. Evaluate.

(a) (3 pts) $(f \circ g)'(2) =$

(b) (3 pts) $(g \circ f)'(2) =$

(5) (**10 pts**) A rectangle has its base on the $x-$axis and two vertices (in the upper half-plane) on the parabola $y = 144 - x^2$. Find the dimensions of the rectangle which ensure that its area is the largest possible.

(6) (**4 pts**) State the **Mean value Theorem for Derivatives**.

(7) (**12 pts**) The volume of a spherical balloon of radius r is given by $V = \frac{4}{3}\pi r^3$.

(a) (6 pts) Find the rate of change of the volume with respect to the radius when the radius is $25cm$.

(b) (6 pts) If the volume of the balloon is decreasing at the rate of $8cm^3$ per minute, what is the rate of change (with respect to time) of the radius when the radius is $25cm$?

(8) (**4 pts**) State the **Extreme Value Theorem**.

(9) (**16 pts**). Consider the function $f(x) = (x+7)(x-1)^3$

(a) (3 pts) Find the critical points of f.

(b) (4 pts) Determine the local extreme values of f.

(c) (3 pts) Determine the intervals where f is increasing.

(d) (3 pts) Determine the intervals where f is concave up.

(e) (3 pts) Determine the inflexion points of f (if any).

(10) (**12 pts**) Use derivatives to evaluate the following limits. In each case, **specify the function being used and its derivative, and the point a at which it is taken**.

(a) $\displaystyle\lim_{h\to 0} \frac{\tan^2(h+\frac{\pi}{4})-1}{h} =$

(b) $\displaystyle\lim_{x\to 1} \frac{x^{2014}-1}{x-1} =$

(c) $\displaystyle\lim_{x\to 0} \frac{e^{5x}-1}{x} =$

(d) $\displaystyle\lim_{x\to 1/\pi} \frac{\ln(\pi x)}{x-1} =$

Nota. $\dfrac{d}{dx}e^x = e^x$ and $\dfrac{d}{dx}\ln|x| = \dfrac{1}{x}$.

Chain Rule: $(f\circ g)'(x) = f'(g(x))\cdot g'(x)$.

Derivative: $f'(a) = \displaystyle\lim_{x\to a}\frac{f(x)-f(a)}{x-a} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$.