Universidad de Puerto Rico
Recinto de Río Piedras
Departamento de Matemáticas
MATE 3151; Examen Departamental III, 18 de noviembre de 2015

Apellidos: ______________ ______________ Nombre ______________
No. Estudiante: ______________ Profesor: ______________ Sección ______________

Instrucciones

Las reglas para este examen son las siguentes.

(1) PARA OBTENER CRÉDITOS, SE DEBE JUSTIFICAR LAS CONTESTACIONES.
(2) NO SE PERMITE USO DE CELULARES.
(3) NO SE PERMITE USO DE CALCULADORAS.
(4) NO SE PERMITE USO DE CUALQUIER OTRO APARATO ELECTRÓNICO.
(5) DEBE TENER DISPONIBLE UNA IDENTIFICAIÓN CON FOTO.

Firma

Problema	Puntuación	Nota
Problema 1	15	
Problema 2	20	
Problema 3	20	
Problema 4	5	
Problema 5	9	
Problema 6	10	
Problema 7	10	
Problema 8	10	
Problema 9	10	
Problema 10[*Bono*]	6	
Total	115	

(1) (15 Pts.) Let $f(x) = 2x + 5$ for $x \in \mathbb{R}$. Let $\mathcal{P}$ be the **uniform partition** of the interval $[1, 5]$ into **four** subintervals.

Notation: for $1 \leq k \leq 4$, $I_k = [x_{k-1}, x_k]$, we write: $\Delta x_k = x_k - x_{k-1}$, , $s_k = \dfrac{x_k + x_{k-1}}{2}$ (=**mid-point** of I_k).

(a) (2 pts) List the points of the partition $\mathcal{P}$:

x_0	x_1	x_2	x_3	x_4

(b) (4 pts) Fill out the following table:

I_k	Δx_k	$f(x_k)$	$f(x_k)\Delta x_k$	$f(x_{k-1})$	$f(x_{k-1})\Delta x_k$	s_k	$f(s_k)$	$f(s_k)\Delta x_k$
Total		XXX		XXX		XXX	XXX	

(c) (2 pts) Evaluate the Riemann sum, $\mathcal{R}(f, P)$, using for $1 \leq k \leq 4$, $c_k =$ the **midpoint** of the interval $[x_{k-1}, x_k]$.

$\mathcal{R}(f, P) =$

(d) (2 pts) Evaluate the Riemann sum $L(f, P)$, using for $1 \leq k \leq 4$, $c_k =$ the **left endpoint** of the interval $[x_{k-1}, x_k]$.

$L(f, P) =$

(e) (2 pts) Evaluate the Riemann sum $U(f, P)$, using for $1 \leq k \leq 4$, $c_k =$ the **right endpoint** of the interval $[x_{k-1}, x_k]$.

$U(f, P) =$

(f) (3 pts) Compute the definite integral $\displaystyle\int_1^5 f(x)dx$

(2) (20 Pts.) Compute the following **indefinite integrals**.

(a) $$\int \frac{x^3+1}{5x^4+20x-9}\,dx =$$

(b) $$\int \frac{\sin(2x)-\cos(2x)}{24\cos(2x)}\,dx =$$

(c) $$\int 12\sin x\cos x\sqrt[5]{2+\cos^2 x}\,dx =$$

(d) $$\int \frac{4x^3+4x^2-5x+7}{24\sqrt[3]{x}}\,dx =$$

(e) $$\int \frac{4x+6}{\sqrt{x^2+3x+1}}\,dx =$$

(3) (20 Pts.) Compute the **definite integrals.**

(a) (5 pts.) $\displaystyle\int_0^1 \frac{\sqrt{x}}{(1+x^{3/2})^9}dx$

(b) (5 pts.) $\displaystyle\int_0^1 (45x^2+15)\sqrt{15x^3+15x}\;dx$

(c) (5 pts.) $\displaystyle\int_0^{\pi/4} 24\tan^5(x)\sec^2(x)dx$

(d) (5 pts.) $\displaystyle\int_4^2 \frac{2e^x+2}{\sqrt{e^x+x+5}}dx$

(4) (5 Pts.) State the **First part of the Fundamental Theorem of Calculus** for a continuous function on $[a, b]$.

(5) (9 Pts.) Evaluate the following **derivatives** using the fundamental theorem of Calculus.

(a) $\dfrac{d}{dx}\left(\displaystyle\int_1^x \frac{5t^2\cos t}{5+t^2}\,dt\right) =$

(b) $\dfrac{d}{dx}\left(\displaystyle\int_e^4 \sqrt{1+2t^3}\,\ln(t^4+5)\,dt\right) =$

(c) $\dfrac{d}{dx}\left(\displaystyle\int_0^{2x} (2t^4+1)^9\,dt\right) =$

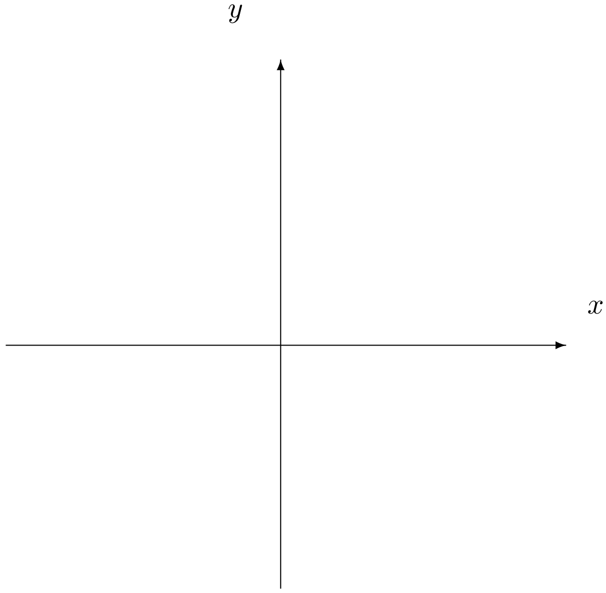
(6) (10 Pts.) Evaluate the following **definite integrals**.

(a) $\int_0^2 u^3(u^2+1)^6 \, du =$
(Hint: set $z = u^2 + 1$)

(b) $\int_0^{\sqrt{\pi}/2} x(1+\tan^2(x^2)) \, dx =$

(7) (10 Pts.) (**Assessment**) Compute the area of the region Ω bounded by the the $x-$axis the line $x + y = 0$ and the line $5x - 6y = 120$.

(a) (2 pts) Sketch the region Ω



(b) (6 pts) Use the definite integral to compute the area of the region Ω.

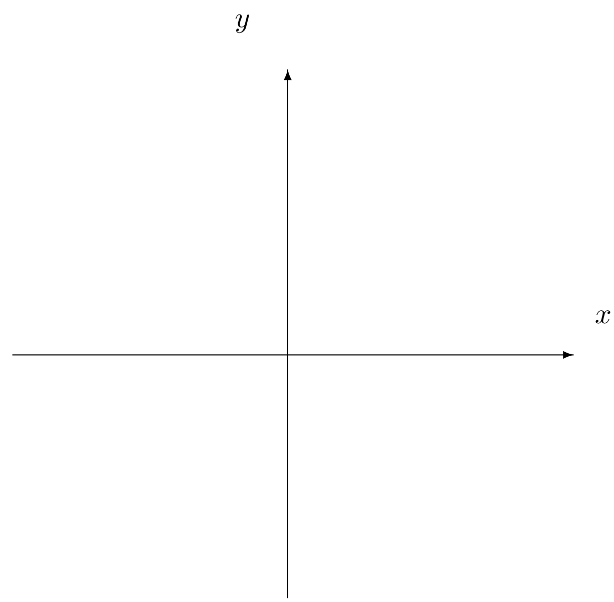
(c) (2 pts) Use the formula from Precalculus (area of a triangle) to compute area of the region Ω.

(8) (10 Pts.) Consider the function $f(x) = 12(x+1)^5$.

(a) (7 pts). Determine the **average value** of $f(x)$ on the interval $[0, 2]$.

(b) (3 pts). Find all numbers c in the interval $(0, 2)$ which satisfy the conclusion of the **Mean Value Theorem for Integrals** for th efunction f.

(9) (10 Pts.) Let Ω be the region bounded by the graphs of the functions $y = x^2$ and $y = 4 - x^2$. **Sketch** the region Ω and calculate the **area** A of Ω.



$A=$

(10) (6 Pts.) [**Bono**] Suppose x and y are related by: $x = \displaystyle\int_{12}^{y} \frac{1}{\sqrt{1+4t^2}} dt$.

Prove that $\dfrac{d^2y}{dx^2}$ is proportional to y and find the constant of proportionality.