Addition and Multiplication Properties of Real Numbers

Axioms of Equality

For all real numbers $a, b,$ and c:

Reflexive Property	$a = a$
Symmetric Property	If $a = b$, then $b = a$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.

Substitution Axiom

If $a = b$, then in any statement involving a we may substitute b for a and obtain another true statement.

Axioms of Addition

Closure	For all real numbers a and b, $a + b$ is a unique real number.
Associative	For all real numbers $a, b,$ and c, $(a+b)+c = a+(b+c)$.
Additive Identity	There exists a unique real number 0 (zero) such that $a + 0 = 0 + a = a$ for every real number a.
Additive Inverses	For each real number a, there exists a real number $-a$ (the opposite of a) such that $a + (-a) = (-a) + a = 0$.
Commutative	For all real numbers a and b, $a + b = b + a$.

Axioms of Multiplication

Closure	For all real numbers a and b, ab is a unique real number.
Associative	For all real numbers $a, b,$ and c, $(ab)c = a(bc)$.
Multiplicative Identity	There exists a unique nonzero real number 1 (one) such that $1 \cdot a = a \cdot 1 = a$.
Multiplicative Inverses	For each nonzero real number a, there exists a real number $\frac{1}{a}$ (the reciprocal of a) such that $a\left(\frac{1}{a}\right) = \left(\frac{1}{a}\right)a = 1$.
Commutative	For all real numbers a and b, $ab = ba$.

The Distributive Axiom of Multiplication over Addition

For all real numbers $a, b,$ and c, $a(b+c) = ab + ac$.

Definition: For all real numbers a, b: