Math 3023 Exam III Topics

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To: Students of Math 3023

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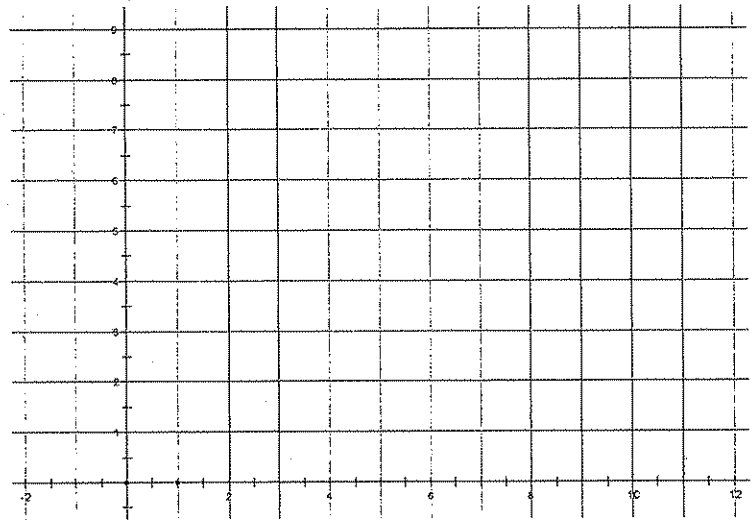
Time: 5:30 P.M.

The students are Responsible for the Following Topics and Procedures

1) **Graphing Piecewise Defined Functions:**

If $f(x)=\begin{cases}(x-1)^2, & \text{if } x\le 2\\ x/2, & \text{if } 2<x<4\\ |x-6|, & \text{if } 4<x\end{cases}$, graph f in the system below.

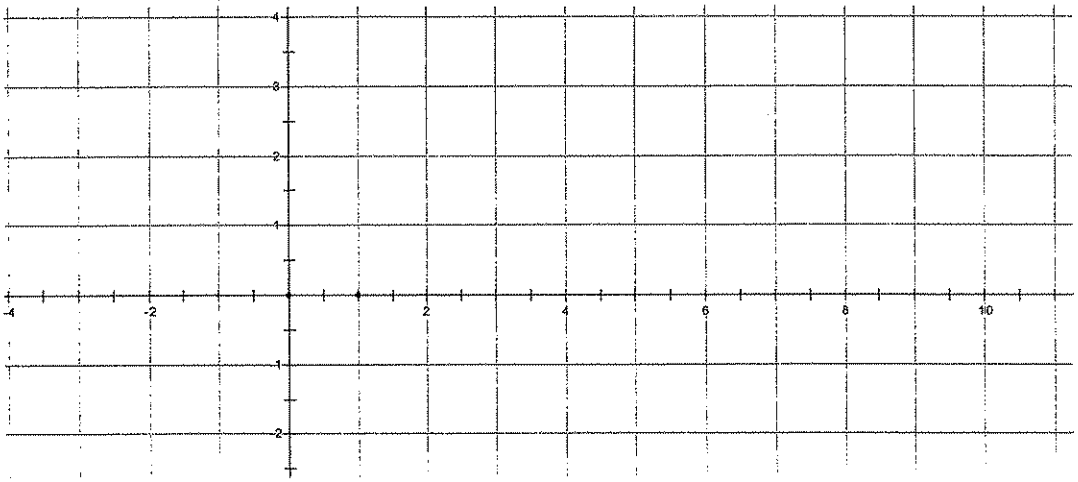


2) **Optimization Problems Involving the Parabola**
   i. Find the dimensions of the rectangle that is inscribed in a right triangle with legs of length 6 and 8 and has the greatest area. Also find that area.
   ii. Find two real numbers whose difference is 36 and whose product is a minimum.

3) **Transformations of Functions**
   a) The student must know the basic transformations:
      i. Scale Change: $S_s(x) = s \cdot x$, includes both horizontal and vertical
      ii. Translation: $T_t(x) = x + t$, includes both horizontal and vertical
      iii. Reflection: $R(x) = -x$, includes both horizontal and vertical
   b) The student must know how to express the transformation of a function f as a composition using the basic transformations. Example: Express $g(x) = 3f(2x)$ as a composition of f and the basic transformations: $g(x) = (S_3 \circ f \circ S_2)(x)$
   c) Exercises: Express as a composition:
      i. $g(x) = -f(x-3)$
      ii. $g(x) = f(-x) - 3$
      iii. $g(x) = 3f(-2x+5) + 7$

d) The student must be able to sketch the transformation of a given function in the same system of coordinates as the original function such as:

Given $f(x)=\sqrt{x}$, sketch the graphs of f and $g(x)=f(2x-3)$ in the same system below.



4) **Testing for Symmetry**
   a) The student must know how to determine if a given function f is symmetric with respect to the $y-$ axis by applying the test $f(-x)=f(x)$
   b) The student must know how to determine if a given function f is symmetric with respect to the origin by applying the test $f(-x)=-f(x)$

5) **Inverse Functions**
   a) The student must know the definition of a one-one function
   b) The student must know the horizontal line test to determine if the graph of a relation is the graph of a one-one function
   c) The student must know the definition of an inverse function
   d) The student must know the Theorem: f one-one $\Rightarrow$ f has a unique inverse
   e) The student must know how to determine the inverse of a one-one function, its domain and its range.

      i. Exercise: If $f(x)=\dfrac{x+3}{x-2}$,
         1. Show f is one-one:
         2. Find $f^{-1}(x)$:
         3. Determine $D_{f^{-1}}$:
         4. Determine $R_{f^{-1}}$:
         5. Verify $(f^{-1}\circ f)(x)=x$:

6) **Arithmetic Sequences and Series**

      i. Exercises:
         1. In the arithmetic sequence $-3,1,5,\cdots$ which term is149?
         2. For what values of x are the terms x, $2x+3$, $4x+18$ consecutive terms of an arithmetic sequence?
         3. Find the sum of the first 20 terms of the arithmetic sequence 4, 6, 8, 10,...
         4. Find the sum of the sequence $-8,-5,-2,\ldots,7$.
         5. Evaluate $\sum_{k=1}^{14}(1-2k)$

6. Find t_{100} and the common difference d and the nth term t_n of the arithmetic sequence, if $t_7 = 31$ and $t_{20} = 96$.
7. A display of cans on a grocery shelf consists of 20 cans on the bottom, 18 cans in the next row, and so on in an arithmetic sequence, until the top row has 4 cans. How many cans, in total, are in the display?
8. How many terms of the arithmetic sequence $-3,2,7,\ldots$ must be added together for the sum of the series to be 116?

7) **Geometric Sequences and Series**

i. Exercises:

1. For what values of x are the terms $x,\ x+2,\ x+3$ consecutive terms of a geometric sequence?
2. Find the 9th term of the sequence $1, \sqrt{2}, 2, \ldots$
3. Determine $s_n = 1 - \frac{1}{3} + \cdots + \left(-\frac{1}{3}\right)^{n-1}$
4. Find the indicated sum $\sum_{k=1}^{n}(0.5)^k$
5. Find the geometric mean of 8 and 6.
6. Insert three geometric means between 1 and 81.

8) **Sign Charts and Graphs of Rational Functions**

Consider the function $g(x) = \frac{x+3}{x-2}$. Determine:

i. its sign chart:
ii. its domain
iii. its range:
iv. any horizontal asymptotes:
v. any vertical asymptotes:
vi. all x intercepts:
vii. any y intercept:
viii. the graph of $g(x)$: