University of Puerto Rico
Department of Mathematics
Mathematics 3023 Exam I September 25, 2013

First Name: _____ Last Name: _____

Student ID: _____ Section: _____

Professor: _____

No credit will be given for any answer without an appropriate justification. No credit will be given for grammatically incorrect mathematical expressions. Your presentation must be neat and organized. (Total Value: 110 pts)

1. (15 pts) For $x \in \mathbb{R}$, let $A = \{x|2x - 3 \geq 0\}$ and $B = \{x|3x + 5 < 0\}$. Express each of the following sets as an interval(s).

   a) $A' \cap B' =$

   b) $A' \cup B =$

   c) $A \cap B' =$

2. (8 pts) Symbolize the following argument using upper case letters. Use the first letter of each underlined word to symbolize the simple statement in which the word appears. If the simple statement repeats, use the same symbol as before to symbolize it. Either use the English or Spanish version.

   If he studies medicine, then he prepares to earn a good living. If he studies the arts, then he prepares to live a good life. If he prepares to earn a good living or he prepares to live a good life, then his college tuition is not wasted. His college tuition is wasted. Therefore, he studies neither medicine nor the arts.

   Si estudia medicina entonces tiene un buen trabajo. Si estudia arte entonces se prepara para vivir una buena vida. Si tiene un buen trabajo o se prepara para vivir una buena vida entonces el dinero invertido en su carrera no es dinero perdido. El dinero invertido en su carrera es dinero perdido. Por lo tanto, no estudió ni medicina ni arte.

3. (5 pts) Construct a truth table of the formula $((p \wedge q) \Rightarrow r) \Rightarrow (p \Rightarrow (q \Rightarrow r))$. Is this formula a *tautology*, a *contingency* or a *contradiction*?

p	q	r					
T	T	T					
T	T	⊥					
T	⊥	T					
T	⊥	⊥					
⊥	T	T					
⊥	T	⊥					
⊥	⊥	T					
⊥	⊥	⊥					

4. (16 pts) Solve the following equalities and inequalities for $x \in \mathbb{R}$. Express your answers as an interval(s).

a) $|2x+5| = |5x-3|$

b) $\dfrac{(x+3)}{(3x+1)(2x-5)} > 0$, by constructing a sign chart first.

c) $|7-5x| < 11$

d) $|6x-7| > 13$

5. (5 pts) Consider the argument form:

$$\begin{array}{l} p \Rightarrow q \\ \underline{q} \\ \therefore p \end{array}$$

Show that this argument form is invalid? Be very clear in you explanation. (Hint: Use a truth table)

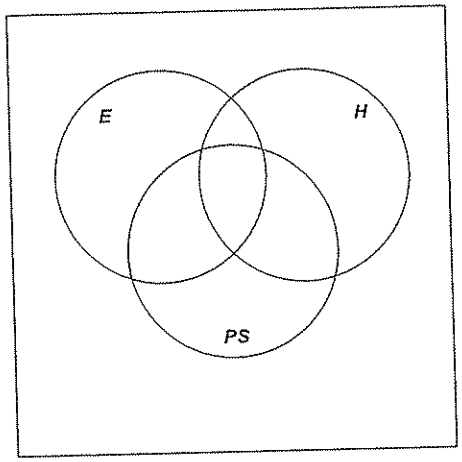
6. (14 pts) A survey of 80 sophomores at a certain western college showed the following:

- 36 take English (E)
- 32 take History (H)
- 32 take Political Science (PS)
- 16 take History and English
- 16 take Political Science and History
- 14 take Political Science and English
- 6 take all three

a) (4 pts) Completely fill in the diagram below with the above data.

How many students—

b) (2 pts) take English and neither of the other two? _____
c) (2 pts) take none of the three courses? _____
d) (2 pts) take History, but neither of the other two? _____
e) (2 pts) take Political Science and History but not English? _____
f) (2 pts) do not take Political Science? _____



7. (10 pts) Suppose you work in a lab. You need a 15% acid solution for a certain test, but your supplier only ships a 10% solution and a 30% solution. You decide to mix 10% solution with 30% solution, to make your own 15% solution. You need 10 liters of the mixture of the 15% acid solution. How many liters of 10% solution and 30% solution should you use? Solve this problem by answering the following items.

   a) (4 pts) Fill in the following table using the above data.

	Liters of sol'n	Percent of Acid	Liters of Acid

   b) (3 pts) Write an equation using your filled table.

   Equation: _____

   c) (3 pts) Solve your equation and determine the liters of 10% solution and 30% solution you must use to make the required 15% solution.

   i) Liters of 10% solution: _____
   ii) Liters of 30% solution: _____

8. (9 pts) Two cars are 500 miles apart and moving directly towards each other. One car is moving at a speed of 100 mph and the other is moving at 70 mph. Assuming that both cars start at the same time, how long does it take for the two cars to meet? Solve this problem by answering the following items.

   a) (3 pts) Fill in the following table using the above data.

	Rate	Time	Distance

   b) (3 pts) Write an equation using your filled table and any other relevant data.

   Equation: _____

   c) (3 pts) Solve your equation for the time it takes for the cars to meet.

   Time it takes the two cars to meet: _____

9. (10 pts) Use the indicated definitions, axioms, or theorems of $\mathbb{R}$ to justify each step in the following proof by filling in the "Reasons" column with the appropriate label(s) from (a)-(l). Please note that the same reason may be used more than once and that not every reason must be used.

Theorem: If $a \in \mathbb{R}$, then $(-1) \cdot a = -a$

Proof:

	Steps	Reasons
1.	$a \cdot (-1) + a = a \cdot (-1) + a$	
2.	$a = a \cdot 1$	
3.	$a \cdot (-1) + a = a \cdot (-1) + a \cdot 1$	
4.	$a \cdot (-1) + a = a \cdot \big((-1) + 1\big)$	
5.	$a \cdot (-1) + a = a \cdot 0$	
6.	$a \cdot (-1) + a = 0$	5; Theorem $a \cdot 0 = 0$
7.	$(a \cdot (-1) + a) + (-a) = 0 + (-a)$	
8.	$a \cdot (-1) + \big(a + (-a)\big) = 0 + (-a)$	
9.	$a \cdot (-1) + 0 = 0 + (-a)$	
10.	$a \cdot (-1) = -a$	
11.	$(-1) \cdot a = -a$	

a) Reflexive Axiom of Equality
b) Transitive Axiom of Equality
c) Substitution Axiom
d) Existence of Additive Identity
e) Existence of Additive Inverse
f) Associative Axiom of Addition
g) Cancellation Theorem of Addition: ("if equals are added to equals the results are equal".)
h) Commutative Axiom of Addition
i) Existence of Multiplicative Identity
j) Existence of Multiplicative Inverse
k) Commutative Axiom of Multiplication
l) Distributive Axiom of Multiplication over Addition

10. (18 pts) Answer the following multiple choice questions.

a) Consider the statement:

If n is divisible by 30, then n is divisible by 2 and by 3 and by 5.

Which of the following statements is the contrapositive of this statement?

(a) If n is not divisible by 30 then n is divisible by 2 or divisible by 3 or divisible by 5.
(b) If n is not divisible by 30 then n is not divisible by 2 or not divisible by 3 or not divisible by 5.
(c) If n is divisible by 2 and divisible by 3 and divisible by 5 then n is divisible by 30.
(d) If n is not divisible by 2 or not divisible by 3 or not divisible by 5 then n is not divisible by 30.
(e) If n is divisible by 2 or divisible by 3 or divisible by 5 then n is divisible by 30.

b) Let ΔABC be a triangle with sides $a, b,$ and c opposite the angles $\angle A$, $\angle B$, and $\angle C$, respectively. Consider the statement:

A sufficient condition that a triangle ΔABC be a right triangle is that $a^2 + b^2 = c^2$.

Which of the following statements is logically equivalent to this statement?

(a) If ΔABC is a right triangle, then $a^2 + b^2 = c^2$.
(b) If $a^2 + b^2 = c^2$, then ΔABC is a right triangle.
(c) If $a^2 + b^2 \neq c^2$, then ΔABC is not a right triangle.
(d) ΔABC is a right triangle only if $a^2 + b^2 = c^2$.
(e) A necessary condition that ΔABC is a right triangle is that $a^2 + b^2 = c^2$.

c) Consider the statement:

You win the game if you know the rules and are not overconfident.

Which of the following statements is the contrapositive of this statement?

(a) A sufficient condition that you win the game is that you know the rules or you are not overconfident.
(b) If you lose the game then you don't know the rules or you are overconfident.
(c) If you don't know the rules or are overconfident then you lose the game.
(d) If you know the rules and are overconfident then you win the game.
(e) A necessary condition that you know the rules or you are not overconfident is that you win the game.

d) Solve the inequality $5 < 1 - 3x \leq 10$.

(a) $\left(\frac{4}{3}, 3\right]$
(b) $\left[-3, -\frac{4}{3}\right)$
(c) None of the above
(d) $\left[-\frac{11}{3}, -\frac{4}{3}\right)$
(e) $\left(-\infty, -\frac{4}{3}\right) \cup [-3, \infty)$

e) Solve the inequality $|1 - x| - 2 \leq 0$ in terms of intervals.

(a) $(-\infty, 3]$
(b) $[-3, 1]$
(c) None of the above
(d) $[-1, 3]$
(e) $(-\infty, -1) \cup [3, \infty)$

f) Solve the inequality $\frac{3x+1}{1-2x} > 0$ in terms of intervals.

(a) $\left(-\frac{1}{3}, \frac{1}{2}\right)$
(b) $\left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{1}{3}, \infty\right)$
(c) None of the above
(d) $\left(-\infty, -\frac{1}{3}\right) \cup \left(\frac{1}{2}, \infty\right)$
(e) $\left(-\frac{1}{3}, \frac{1}{2}\right)$

g) What are the solutions of the inequality $-12 - 10x \leq -8(x + 12)$?

(a) $x \geq -42$
(b) $x \geq 48$
(c) $x \geq 42$
(d) $x \geq 54$
(e) $x \geq -48$