Universidad de Puerto Rico
Departamento de Matemáticas
MATE 3018 – Exam II– October 22, 2008

Apellidos: _____ _____ Nombre _____
No. Estudiante: _____ Profesor: Warma Sección _____

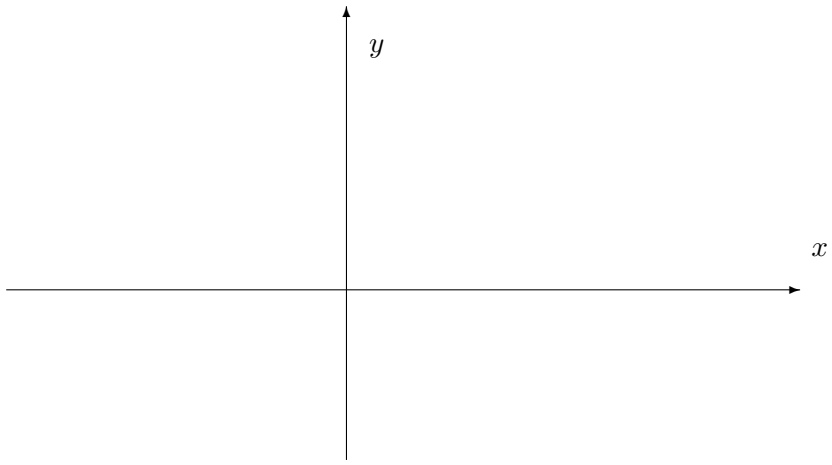
(1) Let f be the function defined on $\mathbb{R}$ by $f(x) = \dfrac{2x+3}{x^2+1}$. Evaluate:

(a) (**2 Pts**) $f(-4)$=

(b) (**2 Pts**) $f(x+3) =$

(c) (**4 Pts**) $\dfrac{f(x+h)-f(x)}{h} =$

(2) (**5 Pts**) Graph $y = 4 - x^2$ and $y - 3 = 4 - (x-2)^2$ in the same coordinate plane.



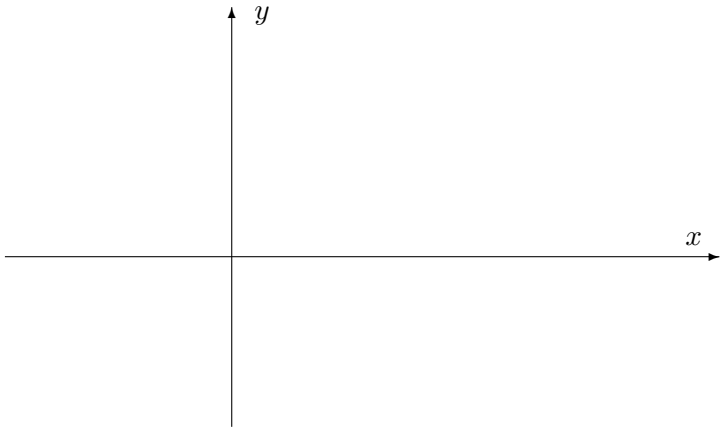
(3) Let $f(x) = \sqrt{4x - 12}$.

(a) (**2 Pts**) Specify the domain and the range of f.

(b) (**2 Pts**) Find $f^{-1}(x)$.

(c) (**2 Pts**) Specify the domain and the range of f^{-1}.

(d) (**4 Pts**) Graph f and f^{-1} in the same coordinate plane.



(4) Find the domain of each of the following functions:

(a) (**2 Pts**) $f(x) = \dfrac{x^2 + 1}{x^2 - 7x + 12}$

(b) (**2 Pts**) $g(x) = \dfrac{1}{\sqrt{8 - 2x^2}}$

(5) Let $f(x) = \dfrac{x+1}{x+2}$ and $g(x) = \dfrac{1}{x}$. Find:

(a) (**2 Pts**) $(f \circ g)(x) =$

(b) (**2 Pts**) $D_{f\circ g} =$

(c) (**2 Pts**) $(g \circ f)(x) =$

(d) (**2 Pts**) $D_{g\circ f} =$

(6) Use the binomial formula to find:

(a) (**2 Pts**) The coefficient of x^7 in the expansion of $(x - \dfrac{2}{\sqrt{x}})^{10}$.

(b) (**2 Pts**) The coefficient of x^5 in the expansion of $(2x + 3)^9$.

(c) (**2 Pts**) The coefficient of x^2 in the expansion of $(\sqrt{x} + \dfrac{3}{\sqrt{x}})^8$.

(d) (**2 Pts**) The third term in the expansion of $(3x - 2)^9$.

(e) (**2 Pts**) The sixth term in the expansion of $(-2x + 3)^8$.

(7) (**2 Pts**) State the principle of mathematical induction.

(8) (**4 Pts**) Use mathematical induction to prove that for all natural numbers n,
$3 + 6 + 9 + \cdots + 3n = \dfrac{3n(n+1)}{2}$.

(9) (**2 Pts**) Define **an arithmetic sequence**.

(10) (**6 Pts**) Find t_{100}**, and the common difference** d **, and the** nth **term** t_n of the arithmetic sequence described by $t_5 = -2$ and $t_{13} = 30$.

(11) (**2 Pts**) Define **a geometric sequence**.

(12) (**4 Pts**) Find t_{16} **and the nth term** t_n of the geometric sequence whose initial term $a = \sqrt{3}$ and with common ratio $q = \sqrt{3}$.

(13) (**4 Pts**) Find x so that $x + 3,\ 2x + 1$, and $5x + 2$ are consecutive terms of an arithmetic sequence.

(14) (**4 Pts**) Find x so that $x,\ x + 2$, and $x + 3$ are consecutive terms of a geometric sequence.

(15) Find each sum.

(a) (**3 Pts**) $\sum_{n=1}^{50}(3n-7) =$

(b) (**3 Pts**) $\sum_{n=1}^{10}\left(\frac{1}{2}\right)^n =$

(16) Given that $\log(x) = -5$, $\log(y) = 7$ and $\log(z) = 8$, evaluate:

(a) (**2 Pts**) $\log(x^5y^9)$=

(b) (**2 Pts**) $\log\left(\sqrt{\frac{x^4}{y^2z^6}}\right)$=

(17) Solve the following equations over the set of real numbers.

(a) (**3 Pts**) $\Big[-4+\ln(3x+2)\Big]^2 = 16$

(b) (**3 Pts**) $\ln\left(\frac{x}{x+2}\right) = 4$

(c) (**3 Pts**) $\log_{16}(x) + \log_4(x) + \log_2(x) = 7$

(d) (**3 Pts**) $4^x - 2^x - 12 = 0$

(18) (**3 Pts**). Find the **inverse** of the function $f(x) = 3 \cdot 2^{4-x}$.

$f^{-1}(x)=$

(19) Consider the functions $f(x) = 2\log_2(x)$ and $g(x) = \log_2(x^2)$. Determine:

(a) (**2 Pts**) $D_f =$

(b) (**2 Pts**) $D_g=$

(c) (**2 Pts**) $f^{-1}(x)=$

(d) (**2 Pts**) $g^{-1}(x)=$

(e) (**4 Pts**) Graph $y = f(x)$ and $y = g(x)$ in the same coordinate plane.