

Universidad de Puerto Rico
Departamento de Matemáticas
MATE 3018 – Exam I– September 16, 2008

Apellidos: _____ Nombre _____
 No. Estudiante: _____ Profesor: Warma Sección OUI

(1) Let $p : 5 - 11 = -6$; $q : 5 \neq \frac{15}{3}$; $r : 7 \leq 10$ and $s : 3 \cdot 4 = -5$. Decide if the following propositions are true or false.

(a) (1 Pts) $p \vee q$:

(b) (1 Pts) $p \rightarrow q$:

(c) (1 Pts) $r \rightarrow p$:

(d) (1 Pts) $p \wedge q'$:

(e) (2 Pts) $[q \wedge (r \rightarrow p)] \rightarrow s'$:

(2) (16 Pts) Consider the open sentences $p \equiv -7x + 1 \geq 21 + 3x$ and $q \equiv -9 + 5x > 2(9 - 2x)$. Find the **solution set** over \mathbb{R} for each of

(a) $CS_p =$	(e) $CS_{p'} =$
(b) $CS_q =$	(f) $CS_{q'} =$
(c) $CS_{p \vee q} =$	(g) $CS_{(p \vee q)'} =$
(d) $CS_{(p \wedge q)} =$	(h) $CS_{(p \wedge q)'} =$

(3) (8 Pts) Let $A = \{x \in \mathbb{R} : x > 9 \text{ or } x < -15\}$ and $B = \{x \in \mathbb{R} : -6 \leq x \leq 7\}$. Find:

(a) $A \cap B =$

(b) $A \cup B =$

(c) $(A \cap B)' =$

(d) $(A \cup B)' =$

(4) (10 Pts) Find the solution set over \mathbb{R} .

(a) $|5x - 16| \geq 8$

(b) $|-11x + 5| \leq 16$

(c) $2 < |x| < 7$

(d) $\frac{3 - 2x}{3x + 5} \leq -1$

(e) $\frac{2x - 3}{x^2 - 3x + 2} \geq 0$

- (5) (a) (2 Pts) The **negation** of $\{x \in \mathbb{R} : x \leq -10 \text{ or } x > -3\}$ is :
- (b) (2 Pts) The **negation** of $\{x \in \mathbb{R} : -3 \leq x < 10\}$ is :
- (6) (2 Pts) State the **negation** of the following sentence: For all $\varepsilon > 0$, there exists $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.
- (7) (4 Pts) State the **contrapositive** of each conditional sentence.
- (a) If $ab = ac$ and $a \neq 0$, then $b = c$.
- (b) If $x^2 - 4 = 0$, then $x = 2$ or $x = -2$.
- (8) (10 Pts) Find an equation for the line with the given properties.
- (a) Parallel to the line $6x + 3y = 7$, containing the point $(-1, 2)$.
- (b) Perpendicular to the line $y + 2x = 2$, containing the point $(-3, 0)$.
- (c) Slope= -2 and y -Intercept= -2 .
- (d) Vertical line and containing the point $(4, -5)$.
- (e) x -Intercept= -4 and y -Intercept= 4 .

(9) (6 Pts) Let $a, b \in \mathbb{R}$ with $a \neq 0$ and consider the points $A = (0, 0)$, $B = (a, b)$ and $C = (13a, 13b)$.

(a) Show that A , B and C are collinear.

(b) Show that B is between A and C .

(10) (4 Pts) If the midpoint M of the line segment AB is $M = (4, 7)$ and $B = (-5, -10)$, find the coordinates of A .

(11) (6 Pts) Find the lengths of the medians of the triangle with vertices at $A = (0, 0)$, $B = (10, 0)$, and $C = (8, 8)$.

(12) (**6 Pts**) Find the center and the radius of the circle given by the equation $x^2 + y^2 - 4x + 6y + 4 = 0$.

(13) (**6 Pts**) Find an equation of the tangent line to the circle $(x - 2)^2 + (y + 3)^2 = 13$ at the point $(4, 0)$.

(14) Find the equation of each parabola.

(a) (**4 Pts**) Vertex at $(0, 0)$ and Focus at $(-4, 0)$.

(b) (**4 Pts**) Focus at $(2, 4)$ and the directrix is the line $x = -4$:

(15) Consider the parabola given by the equation $x^2 + 6x - 4y + 1 = 0$.

(a) (**3 Pts**) Write $x^2 + 6x - 4y + 1 = 0$ in the form $y - k = \frac{1}{4c}(x - h)^2$.

(b) (**1 Pts**) Find the coordinates of the vertex:

(c) (**1 Pts**) Find the coordinates of the focus:

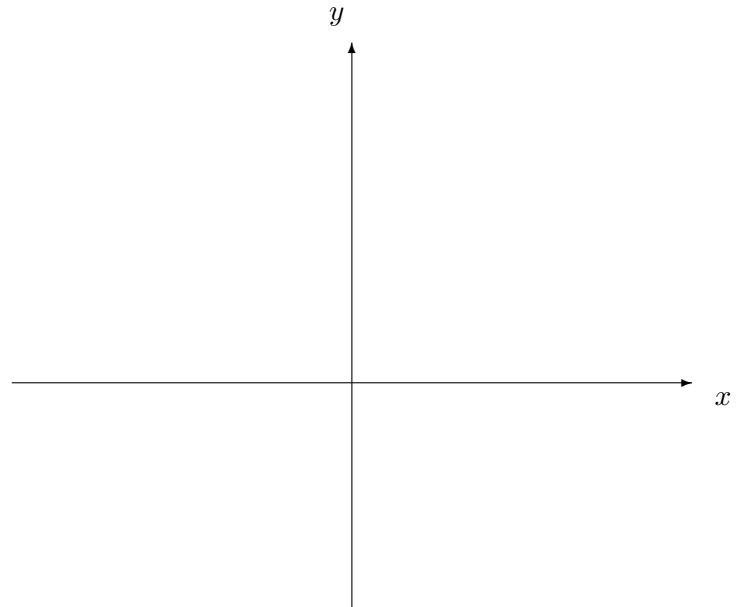
(d) (**1 Pts**) Find the axis of symmetry:

(e) (**1 Pts**) Find the equation of the directrix:

(f) (**1 Pts**) Find the y -intercepts:

(g) (**1 Pts**) Find the x -intercepts:

(h) (**3 Pts**) Sketch the graph of the parabola:



(16) (12 Pts) Find the equation of the circle that contains the point $A = (0, 0)$, $B = (2, 4)$ and $C = (3, 3)$.