

Universidad de Puerto Rico
Departamento de Matemáticas
MATE 3018 – Final Exam – December 12, 2007

Apellidos: _____ Nombre _____
No. Estudiante: _____ Profesor: Warma Sección OU1

(1) For each question **explain** your answer.

(a) (4 Pts) Let $f(x) = 5x^3 - 10$ and $g(x) = 3 - x^2$. Evaluate:
 $(f \circ g)(3) =$

(b) (2 Pts) $\log_2 \left(\frac{1}{32} \right) =$

(c) (2 Pts) $\ln(\sqrt{e^5}) =$

(d) (3 Pts) Write in the form $a + bi$.

$$\frac{2i + 3}{2 - 5i} =$$

(2) Consider the function $f(x) = x^2 - 5x - 10$. Evaluate:

(a) (2 Pts) $f(3) =$

(b) (3 Pts) $\frac{f(x) - f(3)}{x - 3} =$

(c) (3 Pts) $\frac{f(x+h) - f(x)}{h} =$

(3) Given that $t_7 = 28$ and $t_{18} = 83$ are terms of an arithmetic sequence, find:

(a) (4 Pts) $t_{30} =$

(b) (4 Pts) $S_{30} := \sum_{k=1}^{30} t_k =$

(4) Solve the following equations or inequations over the set of real numbers.

(a) (3 Pts) $|2x - 5| \geq 4$

(b) (4 Pts) $[\ln(x)]^2 - 5\ln(x) + 6 = 0$

(c) (3 Pts) $\frac{2 - 4x}{x + 5} \leq 0$

(d) (3 Pts) $4 - x^2 \leq 0$

(5) Find the domain of each of the following functions:

(a) (2 Pts) $f(x) = \frac{x + 1}{x^2 - 4}$

(b) (2 Pts) $g(x) = \frac{1}{\sqrt{4 - x^2}}$

- (6) (6 Pts) Let $P(x) = x^3 - 2x + 4$. Verify that $P(1 + i) = 0$ and find all roots of $P(x) = 0$.
- (7) (4 Pts) Find the equation of the line that is perpendicular to the line $x - 4y = 5$ and passes through the point $(2, -5)$.
- (8) (5 Pts) Find the coordinates of the center and the radius of the circle with equation $x^2 + y^2 + 4x - 7y = -2$.
- (9) Write the **negation** of each sentence.
- (a) (2 Pts) For all $\varepsilon > 0$ there is a $\delta > 0$ such that if $|x - a| < \delta$ then $|f(x) - L| < \varepsilon$:
- (b) (2 Pts) $\{x \in \mathbb{R} : x < -10 \text{ or } x \geq 15\}$:
- (c) (2 Pts) $\{x \in \mathbb{R} : 3 \leq x < 17\}$:
- (10) (9 Pts) Find the exact value of:

(a) $\sin\left(\frac{\pi}{12}\right) =$	(d) $\cos\left(\frac{\pi}{12}\right) =$
(b) $\sin\left(-\frac{11\pi}{4}\right) =$	(e) $\cos(70^\circ)\cos(50^\circ) - \sin(70^\circ)\sin(50^\circ) =$
(c) $\cos\left(\frac{233\pi}{4}\right) =$	(f) $\sin(20^\circ)\cos(70^\circ) + \cos(20^\circ)\sin(70^\circ) =$

(11) Solve the following equations for $0 \leq x < 2\pi$.

(a) **(2 Pts)** $\sin(2x) = \sin(x)$

(b) **(2 Pts)** $2 \sin(x) = \csc(x)$

(12) Express **without using trigonometric or inverse trigonometric functions**.

(a) **(2 Pts)** $\cos(\text{Sin}^{-1}(x)) =$

(b) **(2 Pts)** $\sin(\text{Cos}^{-1}(x)) =$

(13) Consider the triangle ABC with the distances $BC = a = 5$, $AC = b = 5\sqrt{2}$ and $AB = c = 5$.

(a) **(1 Pts)** The angle in A is $\alpha =$

(b) **(1 Pts)** The angle in B is $\beta =$

(c) **(1 Pts)** The angle in C is $\gamma =$

(d) **(2 Pts)** Show that ABC a a right and isosceles triangle. **Explain.**

(14) Consider the function $f(x) = -3 \cos(2x - \pi) + 2$.

(a) (2 Pts) Find the **amplitude**:

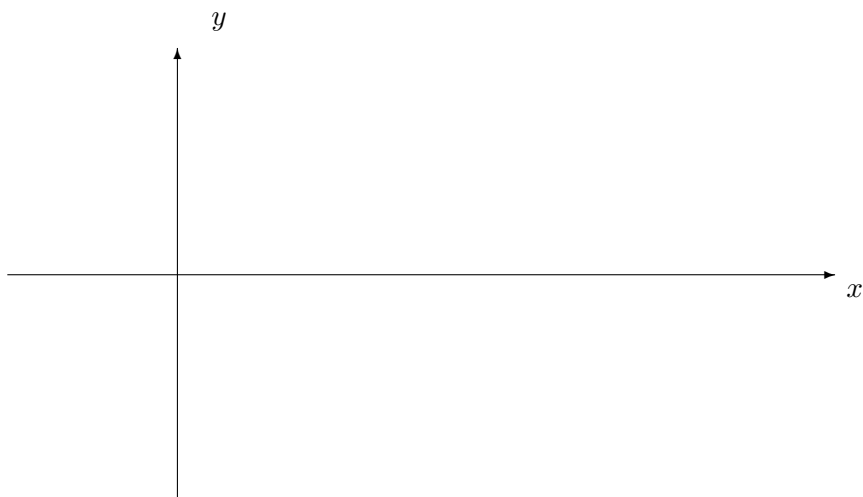
(b) (2 Pts) Find the **period**:

(c) (2 Pts) Find the **phase shift**:

(d) (2 Pts) Determine the **starting point of one cycle** of the graph:

(e) (2 Pts) Determine the **ending point of one cycle** of the graph:

(f) (6 Pts) Find the **five key points** to the graph of $y = -3 \cos(2x - \pi)$ and **graph one cycle** of $y = -3 \cos(2x - \pi)$ and $y = -3 \cos(2x - \pi) + 2$



(15) (4 Pts) Write $y = 2x^2 - 4x + 6$ in the form $y - k = \frac{1}{4c}(x - h)^2$.

(16) (3 Pts) Find the distance between the point $(3, -2)$ and the line $x - y = 2$.