

Universidad de Puerto Rico
Departamento de Matemáticas
MATE 3018 – Exam II– October 17, 2007

Apellidos: _____ Nombre _____
No. Estudiante: _____ Profesor: Warma Sección _____

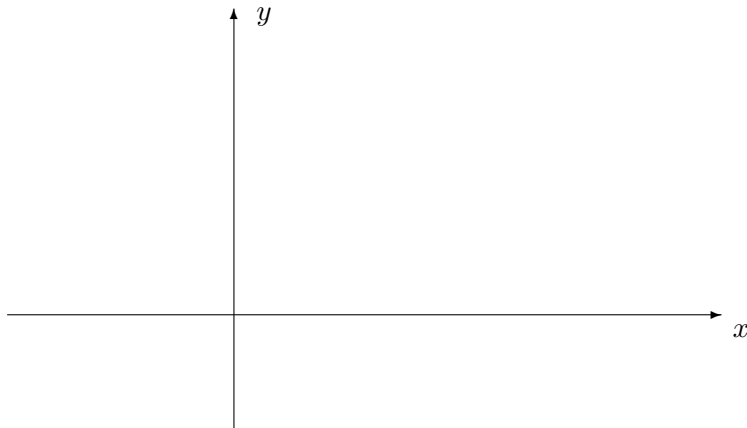
(1) Let f be the function defined on \mathbb{R} by $f(x) = -3x^2 + 5x - 7$. Evaluate:

(a) **(2 Pts)** $f(-3) =$

(b) **(2 Pts)** $f(x - 4) =$

(c) **(4 Pts)** $\frac{f(x + h) - f(x)}{h} =$

(2) **(5 Pts)** Graph $g(x) = |x|$ and $G(x) = 2 + |x - 3|$ in the same coordinate plane.



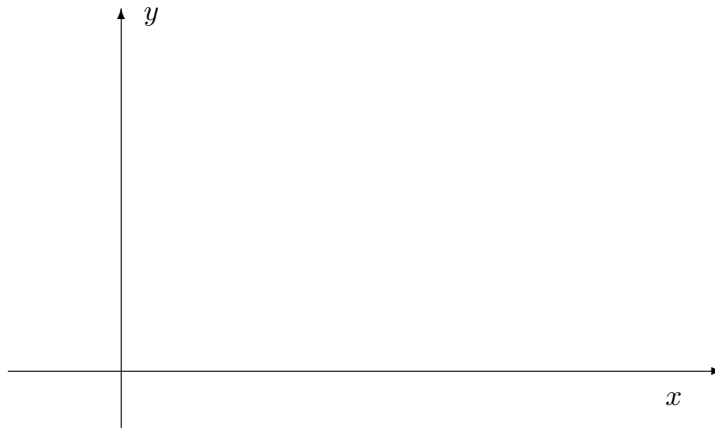
(3) Let $f(x) = \sqrt{x-2}$.

(a) (2 Pts) Specify the domain and the range of f .

(b) (4 Pts) Find $f^{-1}(x)$ and check your result.

(c) (2 Pts) Specify the domain and the range of f^{-1} .

(d) (4 Pts) Graph f and f^{-1} in the same coordinate plane.



(4) Find the domain of each of the following functions:

(a) (2 Pts) $f(x) = \frac{x^2 + 1}{x^2 - 1}$

(b) (2 Pts) $g(x) = \frac{1}{\sqrt{4 - x^2}}$

(5) Let $f(x) = \frac{x-2}{x}$ and $g(x) = \frac{1}{x}$. Find a formula and specify the domain.

(a) (2 Pts) $(f \circ g)(x) =$

(b) (2 Pts) $D_{f \circ g} =$

(c) (2 Pts) $(g \circ f)(x) =$

(d) (2 Pts) $D_{g \circ f} =$

(6) Use the binomial formula to find:

(a) (2 Pts) The coefficient of x^{10} in the expansion of $(x^4 - \frac{1}{x^2})^4$.

(b) (2 Pts) The coefficient of x^4 in the expansion of $(x - \frac{2}{\sqrt{x}})^{10}$.

(c) (2 Pts) The coefficient of x^7 in the expansion of $(2x + 3)^9$.

(d) (2 Pts) The third term in the expansion of $(x - 3)^7$.

(e) (2 Pts) The sixth term in the expansion of $(3x + 2)^8$.

(7) (2 Pts) State the principle of mathematical induction.

- (8) **(5 Pts)** Use mathematical induction to prove that $3 + 5 + 7 + \cdots + (2n + 1) = n(n + 2)$.
- (9) **(2 Pts)** Define an arithmetic sequence.
- (10) **(6 Pts)** Find t_{100} , **and the common difference r** , **and the n th term t_n** of the arithmetic sequence described by $t_7 = 31$ and $t_{20} = 96$.
- (11) **(2 Pts)** Define a geometric sequence.
- (12) **(4 Pts)** Find t_{12} **and the n th term t_n** of the geometric sequence whose initial term $a = \sqrt{2}$ and with common ratio $q = \sqrt{2}$.
- (13) **(4 Pts)** Find x so that $x - 1$, x , and $x + 2$ are consecutive terms of a geometric sequence.
- (14) Find each sum.
- (a) **(3 Pts)** $\sum_{n=1}^{80} (2n - 5) =$
- (b) **(3 Pts)** $\sum_{n=1}^7 \left(\frac{2}{3}\right)^n =$

(15) (**5 Pts**) Suppose that you have just been hired at an annual salary of \$18,000 and expect to receive annual increases of 5%. What will your salary be when you begin your fifth year?

(16) Given that $\log(x) = -2$, $\log(y) = 4$ and $\log(z) = 6$, evaluate:

(a) (**2 Pts**) $\log(x^3y^5) =$

(b) (**2 Pts**) $\log\left(\sqrt{\frac{x^3}{yz}}\right) =$

(17) Solve the following equations over the set of real numbers.

(a) (**3 Pts**) $\left[1 + \ln(3x + 2)\right]^2 = 9$

(b) (**3 Pts**) $\ln(x + 2) - \ln(x + 3) = 4$

(c) (**3 Pts**) $2^{2x} - 12 \cdot 2^x + 32 = 0$

(18) (4 Pts) Find real numbers a and b such that $\ln\left(\frac{x^2 + 6x + 9}{16}\right) = a[\ln(x + 3) + b]$.

$$\ln\left(\frac{x^2 + 6x + 9}{16}\right) =$$

(19) Consider the functions $f(x) = e^{2x-3}$ and $g(x) = \ln(x)$. Determine:

(a) (2 Pts) $(g \circ f)(x) =$

(b) (2 Pts) $D_{g \circ f} =$

(c) (2 Pts) $CV_{g \circ f} =$

(d) (4 Pts) Graph $h(x) = 2^x$ and $(g \circ f)(x)$ in the same coordinate plane.

