

Universidad de Puerto Rico
Departamento de Matemáticas
MATE 3018 – Exam I– September 12, 2007

Apellidos: _____ Nombre _____
 No. Estudiante: _____ Profesor: Warma Sección _____

(1) (6 Pts) Let $p : 3 + 11 = 14$; $q : 3 = \frac{6}{3}$; $r : 7 \geq 10$ and $s : 3 \cdot 4 = 12$. Decide if the following propositions are true or false.

(a) $p \vee q :$

(b) $p \rightarrow q :$

(c) $r \rightarrow p :$

(d) $p \wedge q' :$

(e) $[p \wedge (q \rightarrow r)] \rightarrow s :$

(2) (12 Pts) Consider the open sentences $p \equiv -3x - 1 \leq 15 + 5x$ and $q \equiv -9 + 7x < 2(9 - x)$. Find the **solution set and graph** over \mathbb{R} for each of

(a) $CS_p =$	(d) $CS_{p'} =$
(b) $CS_q =$	(e) $CS_{q'} =$
(c) $CS_{p \wedge q} =$	(f) $CS_{(p \vee q)'} =$

(3) (12 Pts) Let $A = \{x \in \mathbb{R} : x \geq 7 \text{ or } x < -7\}$ and $B = \{x \in \mathbb{R} : -5 \leq x \leq 5\}$. Find and graph:

(a) $A \cap B =$

(b) $A \cup B =$

(c) $(A \cap B)' =$

(d) $(A \cup B)' =$

(e) $A' \cup B' =$

(f) $A' \cap B' =$

(4) (9 Pts) Find the solution set over \mathbb{R} .

(a) $|7x + 13| \geq 4$

(b) $|11x - 5| \leq 3$

(c) $\frac{3 - 2x}{3x + 5} \leq -1$

(5) (a) (**2 Pts**) The **negation** of $\{x \in \mathbb{R} : x < -13 \text{ or } x \geq 7\}$ is :

(b) (**2 Pts**) The **negation** of $\{x \in \mathbb{R} : -5 \leq x < 12\}$ is :

(6) (**4 Pts**) State the **contrapositive** of each conditional sentence.

(a) If $ab = ac$ and $a \neq 0$, then $b = c$.

(b) If $x^2 - 25 = 0$, then $x = 5$ or $x = -5$.

(7) (**9 Pts**) Find an equation for the line with the given properties.

(a) With slope -5 , containing the point $(2, -5)$.

(b) Parallel to the line $4x + 2y = -5$, containing the point $(4, 5)$.

(c) Perpendicular to the line $y + 2x = -7$, containing the point $(-1, 2)$.

(8) (**6 Pts**) Let $a, b \in \mathbb{R}$ with $a \neq 0$ and consider the points $A = (0, 0)$, $B = (a, b)$ and $C = (5a, 5b)$.

(a) Show that A , B and C are collinear.

(b) Show that B is between A and C .

(9) (**4 Pts**) If the midpoint M of the line segment AB is $M = (3, 5)$ and $B = (-3, -7)$, find the coordinates of A .

(10) (**9 Pts**) Find the lengths of the medians of the triangle with vertices at $A = (0, 0)$, $B = (4, 0)$, and $C = (6, 6)$.

(11) (3 Pts) Find the coordinates of the center and the radius of the circle given by the following equation $x^2 + y^2 + 4x - 14y = -28$.

(12) (3 Pts) Find the equation of a circle with center on the line $y = 2x + 1$ and is tangent to the y -axis at the point $(4, 0)$.

(13) (3 Pts) Find an equation of the tangent line to the circle $(x - 2)^2 + (y + 3)^2 = 13$ at the point $(4, 0)$.

(14) (a) (3 Pts) Write $8x - 1 = y^2 - 2y$ in the form $x - h = \frac{1}{4c}(y - k)^2$.

(b) (2 Pts) Find the coordinates of the Vertex:

(c) (2 Pts) Find the coordinates of the focus:

(d) (2 Pts) Find the axis of symmetry:

(e) (2 Pts) Find the equation of the directrix:

(15) Consider the parabola given by the equation $y + 1 = (x + 1)^2$. Find:

(a) (**2 Pts**) The coordinates of the vertex:

(b) (**2 Pts**) The coordinates of the focus:

(c) (**2 Pts**) The axis of symmetry:

(d) (**2 Pts**) The equation of the directrix:

(e) (**2 Pts**) The y -intercepts:

(f) (**2 Pts**) The x -intercepts:

(g) (**3 Pts**) Sketch the graph of the parabola:

