

**UNIVERSITY OF PUERTO RICO
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DEPARTMENT OF MATHEMATICS**

**PROBABILITY AND STATISTICS II
PH.D. QUALIFYING EXAM
SYLLABUS**

Reference: Casella G. and Berger R. *Statistical Inference*. 1st edition, 1990.
Wadsworth & Brooks/Cole (Chapters 2-10)

1. Transformations and Expectations
 - 1.1. Distributions of Functions of a Random Variable
 - 1.2. Expected Values
 - 1.3. Moments and Moment Generating Functions
 - 1.4. Differentiating Under an Integral Sign

2. Common Families of Distributions
 - 2.1 Discrete Distributions
 - 2.2 Continuous Distributions
 - 2.3 Exponential Families
 - 2.4 Location and Scale Families

3. Multiple Random Variables
 - 3.1 Joint and Marginal Distributions
 - 3.2 Conditional Distributions and Independence
 - 3.3 Bivariate Transformations
 - 3.4 Hierarchical Models and Mixture Distributions
 - 3.5 Covariance and Correlation
 - 3.6 Multivariate Distributions
 - 3.7 Inequalities and Identities
 - 3.7.1 Numerical Inequalities
 - 3.7.2 Functional Inequalities
 - 3.7.3 Probability Inequalities
 - 3.7.4 Identities

4. Properties of Random Sample
 - 4.1 Basic Concepts of Random Samples
 - 4.2 Sums of Random Variables from a Random Sample
 - 4.3 Convergence Concepts
 - 4.3.1 Convergence in Probability
 - 4.3.2 Almost Sure Convergence
 - 4.3.3 Convergence in Distribution

- 4.4 Sampling from the Normal Distribution
 - 4.4.1 Properties of the Sample Mean and Variance
 - 4.4.2 The Derived Distributions: Student's t and Snedecor's F
- 4.5 Order Statistics

- 5. Principles of Data Reduction
 - 5.1 The Sufficiency Principle
 - 5.1.1 Sufficiency Statistics
 - 5.1.2 Minimal Sufficient Statistics
 - 5.1.3 Ancillary Statistics
 - 5.1.4 Sufficient, Ancillary, and Complete Statistics
 - 5.2 The Likelihood Principle
 - 5.2.1 The Likelihood Function
 - 5.2.2 The Formal Likelihood Principle
 - 5.3 The Invariance Principle

- 6. Point Estimation
 - 6.1 Introduction
 - 6.2 Methods of Finding Estimators
 - 6.2.1 Method of Moments
 - 6.2.2 Maximum Likelihood Estimators
 - 6.2.3 Bayes Estimators
 - 6.2.4 Invariant Estimators
 - 6.3 Methods of Evaluating Estimators
 - 6.3.1 Mean Squared Error
 - 6.3.2 Best Unbiased Estimators
 - 6.3.3 Sufficiency and Unbiasedness
 - 6.3.4 Consistency
 - 6.4 Other Considerations
 - 6.4.1 Asymptotic Variance of Maximum Likelihood Estimators
 - 6.4.2 Taylor Series Approximations

- 7. Hypothesis Testing
 - 7.1 Introduction
 - 7.2 Methods of Finding Tests
 - 7.2.1 Likelihood Ratio Tests
 - 7.2.2 Invariant Tests
 - 7.2.3 Bayesian Test
 - 7.2.4 Union-Intersection and Intersection-Union Tests
 - 7.3 Methods of Evaluating Tests
 - 7.3.1 Error Probabilities and the Power Function
 - 7.3.2 Most Powerful Tests
 - 7.3.3 Unbiased and Invariant Tests
 - 7.3.4 Locally Most Powerful Tests
 - 7.3.5 Sizes of Union-Intersection and Intersection-Union Tests

- 7.4 Other Considerations
 - 7.4.1 Asymptotic Distribution of LRTs
 - 7.4.2 Other Large-Sample Tests

- 8. Interval Estimation
 - 8.1 Introduction
 - 8.2 Methods of Finding Interval Estimators
 - 8.2.1 Inverting a Test Statistic
 - 8.2.2 Pivotal Quantities
 - 8.2.3 Guaranteeing an Interval
 - 8.2.4 Bayesian Intervals
 - 8.2.5 Invariant Intervals
 - 8.3 Methods of Evaluating Interval Estimators
 - 8.3.1 Size and Coverage Probability
 - 8.3.2 Test-Related Optimality
 - 8.3.3 Invariant Optimality
 - 8.3.4 Other Considerations
 - 8.4 Other Considerations
 - 8.4.1 Approximate Maximum Likelihood Intervals
 - 8.4.2 Other Approximate Intervals

- 9. Decision Theory
 - 9.1 Introduction
 - 9.2 Common Decision Theoretic Analysis
 - 9.2.1 Point Estimation
 - 9.2.2 Hypothesis Testing
 - 9.2.3 Interval Estimation
 - 9.3 Decision Theoretic Bayes Rules
 - 9.3.1 Bayesian Decision Problems
 - 9.3.2 Finding Bayes Rules
 - 9.4 Admissibility of Decision Rules
 - 9.4.1 Comparing Decision Rules
 - 9.4.2 Finding Admissible Rules and Complete Classes
 - 9.4.3 Admissibility of the Sample Mean Under Normality
 - 9.5 Minimax Rules
 - 9.6 Invariant Decision Problems
 - 9.7 Stein's Paradox