UNIVERSITY OF PUERTO RICO
RIO PIEDRAS CAMPUS
DEPARTMENT OF MATHEMATICS

PROBABILITY AND STATISTICS II
PH.D. QUALIFYING EXAM
SYLLABUS

Reference: Casella G. and Berger R. *Statistical Inference*. 1st edition, 1990. Wadsworth & Brooks/Cole (Chapters 2-10)

1. Transformations and Expectations
   - 1.1. Distributions of Functions of a Random Variable
   - 1.2. Expected Values
   - 1.3. Moments and Moment Generating Functions
   - 1.4. Differentiating Under an Integral Sign

2. Common Families of Distributions
   - 2.1 Discrete Distributions
   - 2.2 Continuous Distributions
   - 2.3 Exponential Families
   - 2.4 Location and Scale Families

3. Multiple Random Variables
   - 3.1 Joint and Marginal Distributions
   - 3.2 Conditional Distributions and Independence
   - 3.3 Bivariate Transformations
   - 3.4 Hierarchical Models and Mixture Distributions
   - 3.5 Covariance and Correlation
   - 3.6 Multivariate Distributions
   - 3.7 Inequalities and Identities
     - 3.7.1 Numerical Inequalities
     - 3.7.2 Functional Inequalities
     - 3.7.3 Probability Inequalities
     - 3.7.4 Identities

4. Properties of Random Sample
   - 4.1 Basic Concepts of Random Samples
   - 4.2 Sums of Random Variables from a Random Sample
   - 4.3 Convergence Concepts
     - 4.3.1 Convergence in Probability
     - 4.3.2 Almost Sure Convergence
     - 4.3.3 Convergence in Distribution

- 4.4 Sampling from the Normal Distribution
  - 4.4.1 Properties of the Sample Mean and Variance
  - 4.4.2 The Derived Distributions: Student's *t* and Snedecor's *F*
- 4.5 Order Statistics

5. Principles of Data Reduction
   - 5.1 The Sufficiency Principle
     - 5.1.1 Sufficiency Statistics
     - 5.1.2 Minimal Sufficient Statistics
     - 5.1.3 Ancillary Statistics
     - 5.1.4 Sufficient, Ancillary, and Complete Statistics
   - 5.2 The Likelihood Principle
     - 5.2.1 The Likelihood Function
     - 5.2.2 The Formal Likelihood Principle
   - 5.3 The Invariance Principle

6. Point Estimation
   - 6.1 Introduction
   - 6.2 Methods of Finding Estimators
     - 6.2.1 Method of Moments
     - 6.2.2 Maximum Likelihood Estimators
     - 6.2.3 Bayes Estimators
     - 6.2.4 Invariant Estimators
   - 6.3 Methods of Evaluating Estimators
     - 6.3.1 Mean Squared Error
     - 6.3.2 Best Unbiased Estimators
     - 6.3.3 Sufficiency and Unbiasedness
     - 6.3.4 Consistency
   - 6.4 Other Considerations
     - 6.4.1 Asymptotic Variance of Maximum Likelihood Estimators
     - 6.4.2 Taylor Series Approximations

7. Hypothesis Testing
   - 7.1 Introduction
   - 7.2 Methods of Finding Tests
     - 7.2.1 Likelihood Ration Tests
     - 7.2.2 Invariant Tests
     - 7.2.3 Bayesian Test
     - 7.2.4 Union-Intersection and Intersection-Union Tests
   - 7.3 Methods of Evaluating Tests
     - 7.3.1 Error Probabilities and the Power Function
     - 7.3.2 Most Powerful Tests
     - 7.3.3 Unbiased and Invariant Tests
     - 7.3.4 Locally Most Powerful Tests
     - 7.3.5 Sizes of Union-Intersection and Intersection-Union Tests