

**UNIVERSITY OF PUERTO RICO
RIO PIEDRAS CAMPUS
DEPARTMENT OF MATHEMATICS**

**FUNCTIONAL ANALYSIS II
PH.D. QUALIFYING EXAM
SYLLABUS**

1. Topological vector spaces (introduction, examples, linear mappings, boundedness and continuity, Baire category, the Banach-Steinhaus theorem, the open mapping theorem, the closed graph theorem, bilinear mappings).

2. Geometry of Hilbert spaces (inner product spaces, the Cauchy-Schwarz inequality, the Pythagorean theorem, Hilbert spaces, examples of Hilbert spaces, the orthogonal complement theorem, the Riesz representation theorem, the adjoint of a linear and bounded operator, the existence of orthonormal bases, the dimension of Hilbert spaces).

3. Banach algebras (abstract Banach algebras, the space of multiplicative linear functions, invertible elements, the Gelfand transform, basic properties of spectra, the Gelfand-Mazur theorem, the Gelfand theorem for commutative Banach algebras, the spectral radius formula, the Stone-Weierstrass theorem).

4. C^* -algebras (definition, examples, the multiplier algebra, the unitization of a C^* -algebra, $*$ -homomorphisms, Gelfand's structure theorem for commutative C^* -algebras, functional calculus with normal operators, the spectral mapping theorem).

References:

1. W. Rudin, *Functional Analysis*, Second Edition, International Series in Pure and Applied Mathematics, McGraw-Hill, Inc., 1991.
2. R. G. Douglas, *Banach Algebra Techniques in Operator Theory*, Second Edition, Graduate Texts in Mathematics 179, Springer-Verlag, 1998.
3. G. J. Murphy, *C^* -Algebras and Operator Theory*, Academic Press, Inc., 1990.

Exam format:

There will be five problems in the exam. Each problem is worth 25 points. Only the best four solutions of each student will be counted. The passing score is 60 points or more.