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Each problem is worth a maximum of 2 points. The passing minimum score is 20.

1. Find the Moment Generating Function of an Exponential Random Variable with density

$$f(x|\beta) = \beta\ exp\ (-x\beta),\ x > 0.,\ \beta > 0.$$

and using it get the variance.

2. Let X and Y be Uniform in the Unit Interval $(0,1)$ and independent. Find the density of $W = X - 2Y$.

3. Let $Y_{(1)} = Min(U_1, U_2, \ldots U_n)$ and $Y_{(n)} = Max(U_1, U_2, \ldots, U_n)$, where $U_1, U_2, \ldots, U_n$ are independent Uniform $(0,1)$ and independent. Find the distribution and density of $Y_{(1)}$ and $Y_{(n)}$.

4. Let (X, Y) a two dimensional random variable. Prove that

$$E[X] = E[E[X|Y]].$$

5. Prove the Markov Inequality: *Suppose that X is a random variable such that $Pr(X \geq 0) = 1$. Then for every given number $t > 0$:*

$$Pr(X \geq t) \leq \frac{E(X)}{t}.$$

6. Using the Markov Inequality prove the Chebyshev Inequality,that is: *Let X be a random variable for which $Var(X)$ exists. Then for any number $t > 0$*

$$Pr(|X - E(X)| \geq t) \leq \frac{Var(X)}{t^2}$$

7. Let X_1, X_2, X_3 random variables for which: $Var(X_i) = 30, i = 1, 2, 3$ and $Cov(X_i, X_j) = -1.$, for $i \neq j$. Find $Var(X_1 + X_2 + X_3)$.

8. When a new machine part is selected for installation, it is first inspected. The probability that the part fails the inspection (and consequently is not used) is equal to .05. If the part passes inspection and is used, its lifetime (i.e. the time until it becomes defective) is exponential with a mean of 110 hours. Before an inspection has taken place, what is the probability that the part will last at least 110 hours?

9. Suppose that X_1, X_2 are independent exponentially distributed random variables, both with parameter β, and let $Y = X_1 + 2X_2$. Find the density of Y.

10. A machine has two parts that could fail and have to be replaced. The probability of failure of parts A and B are .1 and .15 respectively. If failures of these parts are independent of each other, what is the probability that at least one of them will fail?

11. A disease affects a 0.05 of the population. There is a clinical test so that the probability of a positive test given that the person has the disease is 0.95 and the probability of a negative test given the person does not have the disease is 0.89. Suppose that the person gets positive in a test. What is the probability that the person is sick?

12. Prove that the sum of ten independent Normal random variables is Normal.

13. The number of claims that an insurance company gets follows a Poisson r.v. with $\lambda = 50$. The amounts of each claim is Uniformly distributed

over $[0, 10]$. What is the probability that the claims are bigger that 300.? (You may use the Normal approximation).

14. You have a Poisson variable with mean 3. What is the probability that the variable is exactly equal to 2 given that it is bigger than 0(zero)?

15. S is a Binomial variable with $n = 1000$ and $p = 0.25$. Approximate the probability that S is bigger that 270.