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Each problem is worth a maximum of 2 points. The passing minimum score is 20.

1. Find the Moment Generating Function of a Poisson Random Variable with
$$f(x|\lambda) = \exp(-\lambda)\frac{\lambda^x}{x!}, \ \lambda > 0, \ x = 0, 1, \ldots$$
and using it get the variance.

2. Let X and Y be Uniform in the Unit Interval $(0, 1)$ and independent. Find the density of $W = 2X - Y$.

3. Let $Y_{(n)} = Max(U_1, U_2, \ldots, U_n)$, where $U_1, U_2, \ldots, U_n$ are independent Uniform $(0, 1)$ and independent. Find the distribution and density of $Y_{(n)}$ and get an expression for its expectation.

4. Prove the Markov Inequality: *Suppose that X is a random variable such that $Pr(X \geq 0) = 1$. Then for every given number $t > 0$:*
$$Pr(X \geq t) \leq \frac{E(X)}{t}.$$

5. Using the Markov Inequality prove the Chebyshev Inequality,that is: *Let X be a random variable for which $Var(X)$ exists. Then for any number $t > 0$*
$$Pr(|X - E(X)| \geq t) \leq \frac{Var(X)}{t^2}$$

6. Let X_1, X_2, X_3 random variables for which: $Var(X_i) = 60, i = 1, 2, 3$ and $Cov(X_i, X_j) = 1.$, for $i \neq j$. Find $Var(X_1 + X_2 + X_3)$.

7. A disease affects a 0.02 of the population. There is a clinical test so that the probability of a positive test given that the person has the disease is 0.94 and the probability of a negative test given the person does not have the disease is 0.84. Suppose that the person gets negative in a test. What is the probability that the person is not sick?

8. Prove that the sum of four independent Normal random variables is Normal.

9. The number of claims that an insurance company gets follows a Poisson r.v. with $\lambda = 100$. The amounts of each claim is Uniformly distributed over $[0, 50]$. What is the probability that the claims are bigger that $2,000$?. (You may use the Normal approximation).

10. You have a Poisson variable with mean 20. What is the probability that the variable is exactly equal to 19 given that it is bigger than 0(zero)?

11. S is a Binomial variable with $n = 200$ and $p = 0.15$. Approximate the probability that S is bigger than 35.

12. A coin is tossed repeatedly. Let X be the random variable for the number of Tails before the first Head is recorded. What are $E(X)$ and $Var(X)$?

13. An insurance company divides its policyholders into low-risk and high-risk classes. For the year, of those in the low-risk class 80% had no claims, 10% had one claim, and 10% had 2 claims. Of those in the high-risk class, 20% had no claims, 40% had one claim, and 40% had two claims. Of the policyholders, 60% were in the low-risk class and 40% in the high-risk class. If a policyholder had two claims in the year, what is the probability that he is in the high-risk class?

14. Suppose that X_1, X_2 are independent Poisson distributed random variables, both with parameter λ, and let $W = X_1 + X_2$. Find the density of W and its expectation.

15. It is assumed that the number of patients checking into a hospital emergency room is Poisson distributed with average of 25 per evening. It is known that 1 in 20 emergency patients ends in death. What is the probability that there will be : i) at least 1 death in a evening? ii) at most 1 death in the evening?