UNIVERSITY OF PUERTO RICO
RIO PIEDRAS CAMPUS
DEPARTMENT OF MATHEMATICS

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Each problem is worth a maximum of 2 points. The passing minimum score is 20.

1. Find the Moment Generating Function of an Exponential Random Variable with density

$$f(x|\beta) = 1/\beta \ exp \ (x/\beta),$$

and using it get the variance.

2. Let X and Y be Uniform in the Unit Interval $(0, 1)$. Find the density of $W = X + 2Y$.

3. Let $Y_{(1)} = Min(U_1, U_2, \ldots U_n)$ and $Y_n = Max(U_1, U_2, \ldots, U_n)$, where $U_1, U_2, \ldots, U_n$ are independent Uniform $(0, 1)$. Find the distribution and density of $Y_{(1)}$ and $Y_{(n)}$.

4. Let (X, Y) a two dimensional random variable. Prove that

$$E[X] = E[E[X|Y]].$$

5. Let X_1, X_2, X_3 random variables for which: $Var(X_i) = 6, i = 1, 2, 3$ and $Cov(X_i, X_j) = -2$. Find $Var(X_1 + X_2 + X_3)$.

6. A rare disease affects a 0.005 of the population. There is a clinical test so that the probability of a positive test given that the person has the disease is 0.98 and the probability of a negative test given the person does not have the disease is 0.91. Suppose that the person gets positive in a test. What is the probability that the person is sick?

7. Prove that the sum of three independent Normal random variables is Normal.

8. The number of claims that an insurance company gets follows a Poisson r.v. with $\lambda = 200$. The amounts of each claim is Uniformly distributed over $[0, 100]$. What is the probability that the claims are bigger that $10,000$. (You may use the Normal approximation).

9. You have a Poisson variable with mean 2. What is the probability that the variable is exactly to 1 given that it is bigger than 0(zero)?

10. S is a Binomial variable with $n = 100$ and $p = 0.25$. Approximate the probability that S is bigger that 27.

11. A single fair die is rolled repeatedly. Let X be the random variable for the number of non-ones rolled before the fifth one is rolled. What are $E(X)$ and $Var(X)$?

12. An insurance company divides its policyholders into low-risk and high-risk classes. For the year, of those in the low-risk class 70% had no claims, 25% had one claim, and 5% had 2 claims. Of those in the high-risk class, 40% had no claims, 40% had one claim, and 20% had two claims. Of the policyholders, 60% were in the low-risk class and 40% in the high-risk class. If a policyholder had two claims in the year, what is the probability that he is in the high-risk class?

13. When a new machine part is selected for installation, it is first inspected. The probability that the part fails the inspection (and consequently is not used) is equal to .02. If the part passes inspection and is used, its lifetime (i.e. the time until it becomes defective) is exponential with a mean of 100 hours. Before an inspection has taken place, what is the probability that the part will last at least 100 hours?

14. Suppose that X_1, X_2 are independent exponentially distributed random variables, and let $Y = X_1 + X_2$. Find the density of Y.

15. A machine has two parts that could fail and have to be replaced. The probability of failure of parts A and B are .1 and .18 respectively. If failures of these parts are independent of each other, what is the probability that at least one of them will fail?