It will be marked the best 5 exercises.

1. Let $X$ be a random variable with moment generating function: $M_X(t); -r < t < r$. Prove that: a)
   $$Pr(X \geq a) \leq \exp(-at)M_X(t), 0 < t < r$$
   and b)
   $$Pr(X \leq a) \leq \exp(-at)M_X(t), -r < t < 0.$$

2. Let $[x_1, \ldots, X_n]$ a random sample from the pdf
   $$f(x|\mu) = \exp[-(x - \mu)], \text{ where } -\infty < \mu < x < \infty.$$
   a) Does this pdf belongs to the Exponential Family? b) Find a complete sufficient statistics.

3. Consider the hierarchical model:
   $$X_i \sim \text{Normal}(\theta_i, \sigma^2), i = 1, \ldots, n, \text{ independent}$$
   and
   $$\theta_i \sim \text{Normal}(\mu, \tau^2), i = 1, \ldots, n, \text{ independent},$$
   where $\sigma^2$ and $\tau^2$ are known. a) Calculate the marginal of the $x_i$'s, i.e integrate the $\theta_i$'s b) Are the $X_i$'s marginally independent?

4. a) Provide the assumptions needed about $f(x|\theta)$to prove that
   $$\int f(x|\theta)(\frac{\partial}{\partial \theta} \log f(x|\theta))^2dx = -\int f(x|\theta)(\frac{\partial^2}{\partial \theta^2} \log f(x|\theta))dx.$$
   b) When this holds, how can the Fisher Information may be defined?
5. Let $Z$ and $W$ iid, Normal Standard. Let $Y = \min(Z, W)$. a) Find the density of $Y$. b) Does this have any relationship with a Chi-Square?

6. Let $[X_1, \ldots, X_n]$, a sample from

$$f(x|\theta) = \frac{1}{\theta}, 0 \leq x \leq \theta, \theta > 0.$$ 

a) Estimate $\theta$ by Maximum Likelihood and by the method of moments.
b) Calculate means and variances of both estimators. Which is better?
c) assume a uniform prior for $\theta$ in the positive line. c.1) Compute the posterior density for $\theta$. c.2) For Quadratic Loss what is the optimal Bayes Estimator?