Universidad de Puerto Rico

Facultad de Ciencias Naturales
Departamento de Matemáticas
Recinto de Río Piedras

Qualifying Exam

Area: Linear Programming Date: Friday, February,11, 2008

Each problem carries 10 points.
General Instructi

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Examen Graduado de Aprovechamiento

Area: **Optimización Lineal** **Fecha:** **miercoles 15 de febrero del 2008**

ons: Answer three of the five problems. In case you try more than 3, indicate clearly which are the three problems that you want to be graded; otherwise the first three in your answer papers will be the ones considered by the grading committe.

1. Solve the following problem by dual simplex method.

$$\textbf{Minimize } -7x_1 + 7x_2 - 2x_3 - x_4 - 6x_5$$

subject to

$$x_i \geq 0 \text{ for every } i.$$

and

$$\begin{aligned} 3x_1 - x_2 + x_3 - 2x_4 &= -3 \\ 2x_1 + x_2 + x_4 + x_5 &= -3 \\ -x_1 + 3x_2 - 3x_4 + x_6 &= 12 \end{aligned}$$

2. (a) (`5 pts`) Show that if a linear inequality in a linear program is changed to equality, the corresponding dual variable becomes free.

   (b) (`5 pts`) Using the Northwest Corner Rule, find basic feasible solutions to transportation problem with following requirements: $\mathbf{a} = (10, 15, 7, 8) \;\; = (8, 6, 9, 12, 5)$.

3. a) (5 points) Let

$$S = \{v_1 = (d_1, 0, 0, 1), v_2 = (0, d_2, 0, 1), v_3 = (0, 0, d_3, 1), (0, 0, 0, 0) \,|\, (d_i, d_j) = 1 \text{ for } i \neq j\}$$

   and let $Conv(S)$ be the convex hull of the set S. Does $Conv(S)$ contain a point with all the coordinates positive integer ? Recall that (a, b) is the greatest common divisor of a and b. **Justify your answer.**

   b) (5 points) Prove that if the vectors $a_1, \ldots, a_m$ are basis for the subspace H of $\mathbf{R}^n$, then the vectors

$$a_1, a_2 \ldots, a_{p-1}, a_q, a_{p+1}, \ldots, a_m$$

   with $a_q \in H$, are a basis if and only if $y_{pq} \neq 0$, where y_{pq} is the coefficient of a_p when a_q is presented as a linear combination of $a_1, \ldots, a_{p-1}, a_p, a_{p+1}, \ldots, a_m$.

4. a) (6 points) Let S be a convex set, H a supporting hyperplane of S, and $T = S \bigcap H$. Prove that every extreme point of T is an extreme point of S.

   b) (4 points) Prove $K = \{\mathbf{x} \in \mathbf{R}^n \,|\, A\mathbf{x} = \mathbf{b},\ \mathbf{x} \geq 0\}$ is a convex set.

5. a) (7 points) Given the linear program in standard form

$$\begin{aligned} \text{minimize} \quad & \mathbf{c}^T\mathbf{x} && (1) \\ \text{subject to} \quad & \mathbf{A}\mathbf{x} = \mathbf{b} && (2) \\ & \mathbf{x} \geq \mathbf{0}, && (3) \end{aligned}$$

   where $\mathbf{A}$ is an $m \times n$ matrix of rank m. Prove that if there is a feasible solution, there is a basic feasible solution.

   b) (3 points) If the convex set K defined by (2) and (3) is nonempty, it has at least one extreme point.