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September 6, 2006 **Linear Programming**

SOLVE EXACTLY THREE OUT OF THE FOLLOWING FIVE PROBLEMS:

1. Prove the second part of the fundamental theorem of linear programming. If there is an optimal feasible solution of the linear programming problem.

$$\begin{aligned} \text{minimize} \quad & c_1x_1 + c_2x_2 + \ldots + c_nx_n \\ \text{subject to} \quad & a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ & a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ & a_{m1}x_1 + a_{m2x_2} + \ldots + a_{mn}x_n = b_m \\ & x_i \geq 0 \text{ for } i = 1, 2, \ldots n \end{aligned}$$

then there is an optimal basic feasible solution.

2.

$$\begin{aligned} \text{minimize} \quad & 4x_1 + x_2 + x_3 \\ \text{subject to} \quad & 2x_1 + x_2 + x_3 = 4 \\ & 3x_1 + 3x_2 + x_3 = 3 \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{aligned}$$

3. Let the standard form of the linear programming problem be written in it's matrix form.

$$\begin{aligned} &\text{minimize} \quad && \overrightarrow{c} \cdot \overrightarrow{x} \\ &\text{subject to} \quad && A\overrightarrow{x} = \overrightarrow{b} \\ & && x_i \geq 0 \text{ for } i = 1, 2, \ldots n \end{aligned}$$

where A is the coefficient matrix, $\overrightarrow{c}$, $\overrightarrow{x} \in E^n$ and $\overrightarrow{b} \in E^m$. Suppose $A = (BD)$ where B is the matrix of the first m columns of A and D is matrix of the last $n - m$ columns of A. Describe the simplex method in the matrix form.

4. Solve the linear inequalities

$$\begin{cases} -2x_1 + 2x_2 & \leq & -1 \\ 2x_1 - x_2 & \leq & 2 \\ -4x_2 & \leq & 3 \\ -15x_1 - 12x_2 & \leq & -2 \\ 12x_1 + 20x_2 & \leq & -1 \end{cases}$$

5. Consider the problem

$$\begin{aligned} &\text{minimize} \quad && 2x_1 + x_2 + 4x_3 \\ &\text{subject to} \quad && x_1 + x_2 + 2x_3 = 3 \\ & && 2x_1 + x_2 + 3x_3 = 5 \\ & && x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{aligned}$$

a. What is the dual problem?

b. Note that $\overrightarrow{\lambda} = (1, 0)$ is a feasible solution for the dual. Starting with this $\overrightarrow{\lambda}$. Solve the primal using the primal-dual algorithm.