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May 1, 2003 **Linear Programming**

SOLVE EXACTLY THREE OUT OF THE FOLLOWING FIVE PROBLEMS:

1. Given a linear programming problem in standard form

$$\text{minimize } c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

$$\text{subject to } A(x_1, x_2 \cdots x_n)^T = (b_1, b_2, , b_m)^T$$

$$x_i \geq 0 \quad 1 \leq i \leq n$$

Prove that: If there is a optimal feasible solution, there is a optimal basic feasible solution. (A is a real $m \times n$ matrix)

2. Solve the following problem.

$$\begin{aligned} \text{minimize} \quad & -2x_1 + 4x_2 + 7x_3 + x_4 + 5x_5 \\ \text{subject to} \quad & -x_1 + x_2 + 2x_3 + x_4 + 2x_5 = 7 \\ & -x_1 + 2x_2 + 3x_3 + x_4 + x_5 = 6 \\ & -x_1 + x_2 + x_3 + 2x_4 + x_5 = 4 \end{aligned}$$

x_1 free, $x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0$

3. In the standard linear programming problem 1, suppose $n > m$ and A has rank m. Suppose the first m columns of A is linearly independent. Let B be the $m \times m$ matrix consisted of the first m columns of A. So $A = [B, D], \overrightarrow{x} = (x_B, x_D), c = (c_B, c_D)$

2 points a) What is the relative cost vector?

3 points b) If we choose $\overrightarrow{x_0} = (B^{-1}\overrightarrow{b}, \overrightarrow{0_D})$ solution, how to determine if it is optimal?

5 points c) If $\overrightarrow{x_0}$ is not optimal, how to determine which column of A to enter and which column of A to leave the basis?

4. For the problem

$$\text{minimize} \quad 2x_1 + x_2 + 4x_3$$

$$\text{subject to} \quad x_1 + x_2 + 2x_3 = 3$$

$$2x_1 + x_2 + 3x_3 = 5$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

3 points a) What is the dual problem.

7 points b) Note that $\lambda = [1, 0]$ is feasible for the dual, starting with this λ. Solve the primal using the primal-dual algorithm.

5. Convert the following problem to a linear program in standard form

$$\text{minimize} \quad |x| + |y| + |z|$$

$$\text{subject to} \quad x + y \leq 1$$

$$2x + z = 3$$