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SOLVE EXACTLY THREE OUT OF THE FOLLOWING FIVE PROBLEMS:

1. Given a linear programming problem in the standard for

$$\begin{aligned} &\text{minimize } \mathbf{c}^T\mathbf{x} \\ &\text{subject to } A\mathbf{x} = \mathbf{b} \\ &\mathbf{x} \geq 0 \end{aligned}$$

Prove that: If there is a feasible solution, there is a basic feasible solution. (A is a $m \times n$ matrix. $\mathbf{c}$ and $\mathbf{x}$ are n dimensional vector. $\mathbf{b}$ is a m-dimensional vector)

2. Using simplex method to solve to following linear programming problem.

maximize $3x_1 + x_2 + 3x_3$ subject to

$$\begin{aligned} &2x_1 + x_2 + x_3 \leq 2 \\ &x_1 + 2x_2 + 3x_3 \leq 5 \\ &2x_1 + 2x_2 + x_3 \leq 6 \end{aligned}$$

$$x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0$$

3. Consider the primal problem in standard form

$$\begin{aligned} &\text{minimize } \mathbf{c}^T\mathbf{x} \\ &\text{subject to } A\mathbf{x} = \mathbf{b} \\ &\mathbf{x} \geq 0 \end{aligned} \tag{1}$$

and its corresponding dual

$$\begin{aligned} &\text{minimize } \lambda^T\mathbf{x} \\ &\text{subject to } \lambda^T A \leq \mathbf{c}^T \end{aligned} \tag{2}$$

show that.

If $\mathbf{x}_0$ and λ_0 are feasible for (1) and (2), respectively, and if $\mathbf{c}^T\mathbf{x} = \lambda^T\mathbf{b}$, then $\mathbf{x}_0$ and λ_0 are optimal for their respective problem.

4. Consider the problem of operating a warehouse of cement. The warehouse has a capacity of C tons, there is a cost of $\$r$ for holding one ton of cement for a month. Suppose the warehouse is originally empty and is required to be empty at the end of 3-month period. Suppose at the beginning of the i-th month, the stock level is x_i tons, u_i tons of cement is bought at the price of $\$q_i$ per ton. At the end of the i-th month, s_i tons of cement is sold at the price of $\$p_i$ per ton. To formulate this problem as a linear programming problem to maximize total profit for the 3-month period.

5. What is the dual of the problem

a) minimize $5x_1 - 3x_2$

subject to
$$\begin{aligned} 2x_1 - x_2 + 4x_3 &\leq 4 \\ x_1 + x_2 + 2x_3 &\leq 5 \\ 2x_1 - x_2 + x_3 &\geq 1 \\ x_1 \geq 0 \ , x_2 \geq 0 \ , x_3 &\geq 0 \end{aligned}$$

b) What is the solution of the dual?