SOLVE EXACTLY THREE OUT OF THE FOLLOWING FIVE PROBLEMS:

1. a. What is the maximum number of regiones defined by \( n \) straight lines in the plane?

b. What is the maximum number of finite regiones defined by \( n \) straight lines in the plane?

2. Let \( x^m = x(x + 1) \cdots (x + m - 1) \)
   \[
   \nabla f(x) = f(x) - f(x - 1)
   \]
   what is \( \nabla (x^m) \)?

3. Which of the following statement is true?
   a. If \( f(N) = O(g(N)) \) then \( g(N) = o(f(N)) \)
   b. For \( k \) large enough, \( N = O(\log^k N) \)
   c. \( N \log N = O(N^{1 + \frac{1}{\log N}}) \)
   d. If \( f(N) = O(T(N)), g(N) = O(T(N)) \) and \( f(N) - g(N) = o(T(N)) \) then \( f(N) = O(g(N)) \) and \( g(N) = O(f(N)) \).
4. Show the result of inserting 16, 15, 14, 13, 12, 11 to the AVL tree

```
      4
     / \  \
    2   6
   / \  / \ \
  1  3 5  7
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For each insertion, give the resulting tree, indicate you need to do single rotation, double rotation or no rotation.

5. Suppose we have 11 memories with address 0 through 10, if we use the hash function $h(x) = x \mod 11$ and the separate chaining algorithm for storing data, give an example of a data of size 6 that result a chain of length 6.