SOLVE EXACTLY THREE OUT OF THE FOLLOWING FIVE PROBLEMS:

1. The following program is applied to an array $a$, $a[i] = 7i \mod 15 - 8$

   $a[i] = 0 \leq i \leq 14$, $a$. length = 15.

   a) What is the result?

   b) What is the running time if the array is of size $a$.length = $n$? Give a complete proof.

   public int number (int[ ] $a$)
   / * 1 */ {
      int number1 = 0;
   / * 2 */ for (int $i = 0; i < a.length; $i++)
   / * 3 */ for (int $j = i; i < a.length; $j++)
   / * 4 */ {
      int number2 = 0;
   / * 5 */ for (int $k = i; k < j; $k++)
   / * 6 */ number2 = number2 + $a[k]$;
   / * 7 */ if (number2 > number1)
   / * 8 */ number1 = number2;
   / * 9 */
   / * 10 */ return number1
   / * 11 */   }

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2. Show the result of each AVL tree when 43, 40, 35, 29, 23, 21 and 20 are inserted to the following AVL tree consecutively.

```
    8
   / \   \   \ 
  5   13
 / \ / \ / \/
1  9 11 17
```

3. Let $D(N)$ be the average depth of the binary search trees. Find $D(N)$ as a function of $N$. (hint: consider the left subtree of size $i$ and right subtree of size $N - i - 1$, let $d(N) = ND(N)$ be the sum of the depth of all nodes of the tree, so $d(N) = d(i) + d(N - i - 1) + N - 1$.

4. What is the minimum number of nodes of an AVL tree of height 5? Justify your answer.
5 a). Use quadratic probing to insert 91, 28, 31, 58, 61 to the address 0 to 9. Show the content of each address after each insertion. Where \( \text{hash}(x) = x \mod 10 \), and \( f(i) = i^2 \).
b. Use double hashing to insert the same data of 5a to the same address. Show the content of each address after each insertion.

With $hash_2(x) = 7 - (x \mod 7)$