SOLVE EXACTLY THREE OUT OF THE FOLLOWING FIVE PROBLEMS:

1. Suppose $T_1(N) = O(f(N))$ and $T_2(N) = O(f(N))$ which of the following are true?

   a. $T_1(N) + T_2(N) = O(f(N))$
   
   b. $T_1(N) - T_2(N) = o(f(N))$
   
   c. $\frac{T_1(N)}{T_2(N)} = O(1)$
   
   d. $T_1(N) = \Theta(T_2(N))$
   
   e. $T_1(N) = O(T_2(N))$
   
   f. $T_1(N) = O(T_2(N))$
   
   g. $T_1(N) \cdot T_2(N) = O(f(N) \cdot f(N))$
   
   h. $(T_1(N) + T_2(N))/N = o(f(N))$
   
   i. $T_1(N) = \Omega(T_2(N))$
   
   j. $f(N) = \Omega(T_1(N))$
2. For the following program segment, give an analysis of running time (in Big O notation).

```c
sum = 0;
for (i = 0; i < n; i++)
  for (j = 0; j < n * n; j++)
    if ((j % i) == 0)
      sum +=;
```

where \( j \% i \) is the remainder of \( j \) divided by \( i \).

3. Give the diagram of the AVL trees when 30 29 28 27 is inserted into the following AVL tree

```
     4
    / \ /
   2 7 / /
  / \ / / /
1 3 6 40 35 47
```

4. Give a flow chart of the Depth-First Search algorithm of a graph.

5. Suppose a binary tree has leaves \( l_1, l_2 \cdots l_m \) at depth \( d_1, d_2 \cdots d_m \), respectively. Prove that

\[
\sum_{i=1}^{M} 2^{-l_i} \leq 1
\]

and determine when the equality is true.